6-43 (orthogonality of eigenfunctions)

- The ground state of a simple harmonic oscillator is

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

where I have included the appropriate normalization factor.

- The first excited state (which is incorrect in your textbook!) is

$$\psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \sqrt{\frac{2m\omega}{\hbar}}\; x\, e^{-m\omega x^2/2\hbar}$$

- These energy eigenstates are said to be "orthogonal" if

$$\int_{-\infty}^{\infty} \psi_0(x)\,\psi_1(x)\,dx = 0$$

This is analogous to vectors being "orthogonal" if their dot product is zero. Anyhow, doing the integral...

$$\int_{-\infty}^{\infty} C\, e^{-m\omega x^2/2\hbar}\, x\, e^{-m\omega x^2/2\hbar}\, dx$$

$$= \int_{-\infty}^{\infty} C\, x\, e^{-Ax^2}\, dx = 0 \quad \text{(using Wolfram Alpha).}$$

- So they are orthogonal. This is true of all of the different energy eigenstates.