TL 6-37 Simple harmonic oscillator

Assume SHO in ground state so

$$\psi_0(x) = C_0 e^{-m\omega x^2/2\hbar}$$

a) Find normalization constant, C_0

$$\int_{-\infty}^{\infty} C_0^2 e^{-2m\omega x^2/2\hbar} = 1$$

$$2C_0^2 \int_0^{\infty} e^{-ax^2} = 1 \quad , \quad a = m\omega/\hbar$$

$$C_0^2 = \frac{1}{2I_0} \quad , \quad 2I_0 = \frac{2\sqrt{\pi}}{2\sqrt{a}} = \sqrt{\frac{\pi\hbar}{m\omega}}$$

$$C_0^2 = \sqrt{\frac{m\omega}{\pi\hbar}} \quad , \quad \boxed{C_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}$$

b) Find $\langle x^2\rangle = \int_{-\infty}^{\infty} C_0^2 e^{-m\omega x^2/\hbar} x^2 dx = 2C_0^2 \int_0^{\infty} e^{-ax^2} x^2 dx$

$$= 2\sqrt{\frac{m\omega}{\pi\hbar}}\left[\frac{1}{4}\sqrt{\frac{\pi}{a^3}}\right] = \frac{1}{2}\sqrt{\frac{m\omega}{\hbar}\frac{\hbar^3}{m^3\omega^3}}$$

$$\langle x^2\rangle = \frac{1}{2}\sqrt{\frac{\hbar^2}{m^2\omega^2}} = \boxed{\frac{\hbar}{2m\omega}}$$

c) $\langle V(x) = \frac{1}{2}m\omega^2 x^2\rangle = \quad \langle V(x)\rangle = \frac{1}{2}\cancel{m}\omega^{\cancel{2}}\frac{\hbar}{2\cancel{m\omega}}$

$$\boxed{\langle V(x)\rangle = \frac{\hbar\omega}{4}}$$