TL 5-17 (Harmonic waves)

- Two harmonic waves travel along a wire

$$y_1 = 0.002 \cos(8.0x - 400t)$$
$$y_2 = 0.002 \cos(7.6x - 380t)$$

$$\left\{ \begin{array}{l} y_0 = 0.002 \\ k_1 = 8.0 \quad , \quad k_2 = 7.6 \\ \omega_1 = 400 \quad , \quad \omega_2 = 380 \end{array} \right.$$

a) This can be expressed as

$$\boxed{y(x,t) = 2y_0 \cos\left(\tfrac{1}{2}\Delta k x - \tfrac{1}{2}\Delta\omega t\right)\cos(\bar{k}x - \bar{\omega}t)}$$

where $\Delta k = k_2 - k_1 = -0.4$

$\Delta\omega = \omega_2 - \omega_1 = -20$

$\bar{k} = \frac{1}{2}(k_1 + k_2) = 7.8$

$\bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2) = 390$

b) phase velocity is $v_p = \lambda f = \frac{\omega}{k} = \frac{\bar{\omega}}{\bar{k}} = \frac{390}{7.8} = 50 \text{ m/s}$

$$\boxed{v_p = 50 \text{ m/s}}$$

c) group velocity is $v_g = \frac{\partial\omega}{\partial k} = \frac{\Delta\omega}{\Delta k} = \frac{-20}{-0.4} = 50$

$$\boxed{v_g = 50 \text{ m/s}}$$

This is a non-dispersive medium.