TL 4-16 — Quantization of gravitational orbit

- The earth orbits the sun at radius $r = 1.50 \times 10^{11}\,\mathrm{m}$
- What "quantum number" would correspond to this orbital radius?
- Looking back at how the Bohr model was derived...

$$v^2(r) = \frac{G\cancel{m}M}{\cancel{m}r} \quad , \quad \ell = rmv = n\hbar$$

$$n^2\hbar^2 = r^{\cancel{2}}m^2\frac{GM}{\cancel{r}} \quad , \quad n^2 = \frac{rGm^2M}{\hbar^2}$$

- So this is the quantum # n for the earth, mass m, orbiting the sun, mass M, at radius r. Plugging in numbers...

$$\boxed{n = 2.53 \times 10^{74}}$$

- How much energy would be released in a transition to $(n-1)$?

The orbital energy of the planet at radius r is given by

$$E_r = -\frac{1}{2}G\frac{Mm}{r} = -\frac{1}{2}G\frac{Mm}{\left[n^2\hbar^2/Gm^2M\right]}$$

$$E_n = -\frac{1}{2}\frac{G^2M^2m^3}{n^2\hbar^2}$$

$$E_n = -E_0\frac{1}{n^2} \quad , \quad \text{where } E_0 = \frac{1}{2}\frac{G^2M^2m^3}{\hbar^2}$$

The binding energy of $n=1$ orbit
$$\left\{\begin{aligned} E_0 &= 1.69 \times 10^{182}\ \text{Joules} \\ E_0 &= 1.05 \times 10^{201}\ \text{eV} \end{aligned}\right.$$

- See next page....

- What would be the energy change associated with a drop from (n) to $(n-1)$?

$$\Delta E = -E_0\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$$ ← lets make a common denominator

$$= -E_0\left(\frac{n^2}{(n-1)^2 n^2} - \frac{(n-1)^2}{n^2(n-1)^2}\right)$$

$$= -E_0\left(\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right)$$

and simplify

$$= -E_0\left(\frac{n^2-n^2-1+2n}{n^2(n-1)^2}\right)$$

$$= -E_0\left(\frac{2n-1}{n^2(n-1)^2}\right)$$

- For huge n (like we have), this is approximately

$$\Delta E \overset{\text{approx}}{\approx} -E_0\left(\frac{2n}{n^2 n^2}\right) = -E_0\left(\frac{2}{n^3}\right)$$

- Plugging in our values of E_0 & n from above, we get

$$\cancel{\Delta E \approx 1.33 \times 10^{-108}\ \text{Joules}}$$

$$\cancel{\approx 8.3 \times 10^{-126}\ \text{eV}}$$

oops

$$\boxed{\begin{aligned}\Delta E &\approx 2.087 \times 10^{-41}\ \text{J} \\ \Delta E &\approx 1.303 \times 10^{-22}\ \text{eV}\end{aligned}}$$

- If a "graviton" is emitted, its frequency will be

$$\lambda = \frac{hc}{\Delta E} = \boxed{9.5 \times 10^{15}\ \text{meters} = \lambda}$$

- The radius of the new orbit?

• The radius of the $(n-1)$ orbit, to which the earth has fallen, would be given by

$$r_{n-1} = \frac{(n-1)^2 \hbar^2}{G m^2 M} \approx \frac{n^2 \hbar^2}{G m^2 M} - \cancelto{0}{\frac{2n\hbar^2}{G m^2 M}} + \cancelto{0}{\frac{\hbar^2}{G m^2 M}}$$

$$r_{n-1} \approx r_n$$