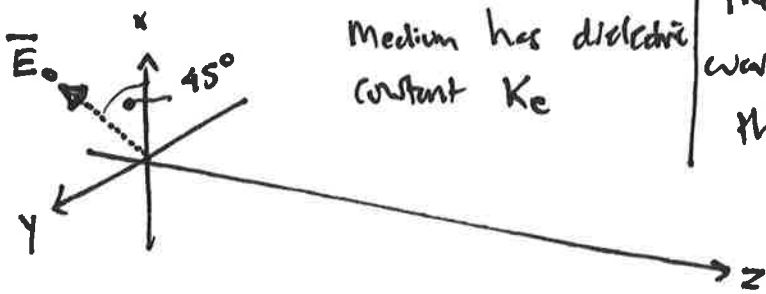
BB. 6.4 (Monochromatic Plane wave in a dielectric)



Medium has dielectric constant K_e

The linearly polarized wave is oriented such that E makes a 45° angle w.r.t. the x-axis.

- The wave has freq. f. It propagates along the z-axis.

$$\bar{E} = \hat{x}\left[\frac{E_0}{\sqrt{2}} e^{i(kz-\omega t)}\right] + \hat{y}\left[\frac{E_0}{2} e^{i(kz-\omega t)}\right]$$

$$\bar{E} = \frac{E_0}{\sqrt{2}} e^{i(kz-\omega t)}\left[\hat{x}+\hat{y}\right]$$

- The wave vector and frequency ω are related by

$$v = \frac{\omega}{k} \Rightarrow k = \frac{\omega}{c} n = \frac{\omega}{c}\sqrt{K_e}$$

(See formulae 6.43 & 6.44). Also $\omega = 2\pi f$, so

$$\bar{E} = \frac{E_0}{\sqrt{2}}\left[\hat{x}+\hat{y}\right] e^{i\left(\frac{2\pi f}{c}\sqrt{K_e}\, z - 2\pi f t\right)}$$

$$\boxed{\bar{E} = \frac{E_0}{\sqrt{2}}\left[\hat{x}+\hat{y}\right] e^{i2\pi f\left(\frac{\sqrt{K_e}}{c} z - t\right)}}$$

- This is the eqn. for the electric field.

What about the magnetic field?

- The magnetic field is given by

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$i\bar{k} \times \bar{E} = +i\omega \bar{B}$$

$$\bar{B} = +\frac{k}{\omega}(\hat{k} \times \bar{E})$$

- The magnitude of $\bar{B}$ is given by

$$\frac{B}{E} = \frac{k}{\omega} = \frac{\omega\sqrt{K_e}}{\omega c} = \frac{\sqrt{K_e}}{c}$$

- The direction of $\bar{B}$ is given by

$$\hat{B} = +\underbrace{\hat{k}}\times\hat{E}$$

$$= +\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} = +\hat{x}\left(\frac{-1}{\sqrt{2}}\right) + \hat{y}\left(\frac{1}{\sqrt{2}}\right)$$

$$\hat{B} = \frac{-1}{\sqrt{2}}(\hat{x} - \hat{y})$$

- So the magnetic field is

$$\boxed{\bar{B} = \frac{\sqrt{K_e}\,E_0}{\sqrt{2}\,c}\left[-\hat{x} + \hat{y}\right] e^{i2\pi f\left(\frac{\sqrt{K_e}}{c}z - t\right)}}$$

- What is the rate of energy flow per unit area?

I will use the definition of the Poynting vector

$$\boxed{\bar{S} \equiv \frac{1}{\mu_0}(\bar{E} \times \bar{B})} = c^2 \epsilon_0 (\bar{E} \times \bar{B})$$

The time average, for a harmonic wave has magnitude:

$$\boxed{\langle|\bar{S}|\rangle = \text{Intensity} = \frac{1}{2}\frac{1}{\mu_0} E_0 B_0}$$

$$\boxed{\langle|\bar{S}|\rangle = \frac{1}{2}\frac{1}{\mu_0}\frac{\sqrt{K_e}}{c} E_0^2} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}\sqrt{K_e}\, E_0^2$$

$$\langle|\bar{S}|\rangle = \frac{1}{2}\frac{\sqrt{\epsilon}}{\sqrt{\mu_0}} E_0^2 \quad (\text{since } \epsilon = K_e \epsilon_0)$$

- What is the wave eqn?

(i) $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

(ii) $\bar{\nabla} \times \left(\frac{\bar{B}}{\cancel{K_m}}\right) = \mu_0 \epsilon_0 K_e \frac{\partial \bar{E}}{\partial t} + \cancel{\mu_0 \sigma} \bar{E}$ 0 dielectric

$K_m \to 1$ non magnetic

(i) $\bar{\nabla} \times \bar{\nabla} \times E = -\bar{\nabla} \times \frac{\partial \bar{B}}{\partial t}$

$\bar{\nabla}(\bar{\nabla} \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}(\bar{\nabla} \times B)$ $\mu_0 \epsilon_0 K_e \frac{\partial E}{\partial t}$

$$\boxed{\nabla^2 \bar{E} = \underbrace{\mu_0 \epsilon_0 K_e}_{\epsilon} \frac{\partial^2 E}{\partial t^2}}$$