BB 6.3 (Standing wave in a dielectric material)

- A (standing) wave solution to maxwell's equations is given by $\bar{E}(\bar{r},t) = \bar{E}_0 \cos(\underbrace{\sqrt{6}}_{k} z)\cos(\underbrace{6\times10^{10}}_{\omega} t)$

- The refractive index can be written as

$$n = n_r - j n_i = \frac{c}{\omega}(k_r - j k_i)$$

- In our case $k_r = \sqrt{6}$ and $k_i = 0$, so $n_r = \frac{c k_r}{\omega}$

- Thus $n = n_r = \frac{(3e10)(\sqrt{6})}{6e10} = \boxed{\frac{\sqrt{6}}{2} \approx 1.225}$

- The magnetic field amplitude is given by $\frac{B_0}{E_0} = \frac{k}{\omega}$

- So $B_0 = E_0 \frac{\sqrt{6}}{6e10} \approx \boxed{4.08\times10^{-11} E_0}$

- Lets use Maxwell's equations to find the function $\bar{B}(z,t)$

- Faraday's law says $\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$, the same in a vacuum or in matter.

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \cancel{\partial_y} & \partial_z \\ E_x & E_y & \cancel{E_z} \end{vmatrix}$$

Since $E_z = 0$, and since $\bar{E} = \bar{E}(\cancel{x},\cancel{y},z,t)$

$$\bar{\nabla}\times\bar{E} = -\partial_z E_y \hat{x} + \partial_z E_x \hat{y}$$

- Plugging in our $\bar{E}$ gives

$$-\partial_z E_y \hat{x} + \partial_z E_x \hat{y} = k \sin(kz)\cos(\omega t)\{E_{oy}\hat{x} - E_{ox}\hat{y}\}$$

- Then, since $\nabla \times \bar{E} = -\partial \bar{B}/\partial t$, we integrate w.r.t. time: to find $\bar{B}$

$$\bar{B} = -\int dt \left(k \sin(kz)\cos(\omega t)\right)\left(E_{oy}\hat{x} - E_{ox}\hat{y}\right)$$

$$\bar{B} = -\frac{k}{\omega}\sin(kz)\sin(\omega t)\left\{E_{oy}\hat{x} - E_{ox}\hat{y}\right\}$$

$$\boxed{\bar{B} = \left(\frac{n}{c}\right)\sin(kz)\sin(\omega t)\left\{E_{ox}\hat{y} - E_{oy}\hat{x}\right\}}$$

where $n = 1.225$

- Note it is 90° out of phase with $\bar{E}$, in both space (z-coordinate) and time (t-coord).