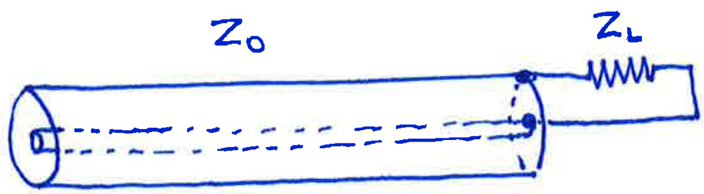
BB 5.2 (Transmission of energy down a coaxial cable.)

- In this problem, we will explore how the type of load (e.g. antenna, speaker, resistor, short) affects the transmission of energy down a particular RF transmission line: the coaxial cable.
- In particular, we will look at this situation:



- The coax has a characteristic impedance Z_0. The load has impedance Z_L. We will consider 2 cases: $Z_L = Z_0$ and $Z_L = 0$. If $Z_L = Z_0$, the load is "impedance matched" to the coax, and there will be no reflected energy. If $Z_L = 0$, the inner & outer conductors are shorted, and the load will act like a mirror, reflecting all the energy sent down the coax.
- We will plot $S(z,t)$, the poynting vector (in $\frac{\text{Watts}}{\text{m}^2}$) at various points along the coax for various times. (in each case).

- Generally speaking, there will be a traveling voltage (and current) wave propagating down the coax. There may also be a reflected voltage (and current) wave. So lets take

$$V(z,t) = \underbrace{V_i e^{i(\omega t - kz)}}_{\text{incident}} + \underbrace{V_r e^{i(\omega t + kz)}}_{\text{reflected}} \qquad \text{(Eq. 5.46)}$$

$$I(z,t) = \overbrace{I_i e^{i(\omega t - kz)}}^{\text{incident}} + \overbrace{I_R e^{i(\omega t + kz)}}^{\text{reflected}}$$

- If we take the load to be at $z=0$, then we have

$$V_L = V(0,t) = (V_i + V_r)\, e^{i(\omega t)}$$

$$I_L = I(0,t) = (I_i + I_R)\, e^{i(\omega t)}$$

- The characteristic impedance of the transmission line is defined as the ratio of voltage to current

$$Z_0 \equiv \frac{V}{I}$$

- Using our voltage and current equations, and (Eqs 5.33), we can show that $I_i = \frac{V_i}{Z_0}$ and $I_R = \frac{-V_R}{Z_0}$. Thus

$$I_L = \left(\frac{V_i}{Z_0} - \frac{V_R}{Z_0}\right) e^{i\omega t}$$

- This gives $Z_L = \frac{V_L}{I_L} = Z_0\left(\frac{V_i + V_r}{V_i - V_r}\right)$

Inverting gives $\boxed{\frac{V_r}{V_i} = \frac{Z_L - Z_0}{Z_L + Z_0}}$ (Eq 5.49) Similarly $\boxed{\frac{I_r}{I_i} = \frac{Z_0 - Z_L}{Z_0 + Z_L}}$ (Eq 5.50)

- We can use eqs. 5.49 & 5.50 to find out the amplitudes of the reflected voltage (or current) waves.

- If $Z_L = Z_0$ then $\frac{V_R}{V_i} = 0 \Longrightarrow$ no reflected voltage

$\frac{I_r}{I_i} = 0 \Longrightarrow$ no reflected current

- So: our voltage and current in the coax looks like

$$V(z,t) = V_i \, e^{i(\omega t - kz)}$$

$$I(z,t) = \left(\frac{V_i}{Z_0}\right) e^{i(\omega t - kz)}$$

- The poynting vector $S(z,t) = \frac{\text{Power}(z,t)}{\text{area}}$ where

Power: $P(z,t) = (\mathrm{Re}\,V)(\mathrm{Re}\,I)$ (Eqn. 5.42)

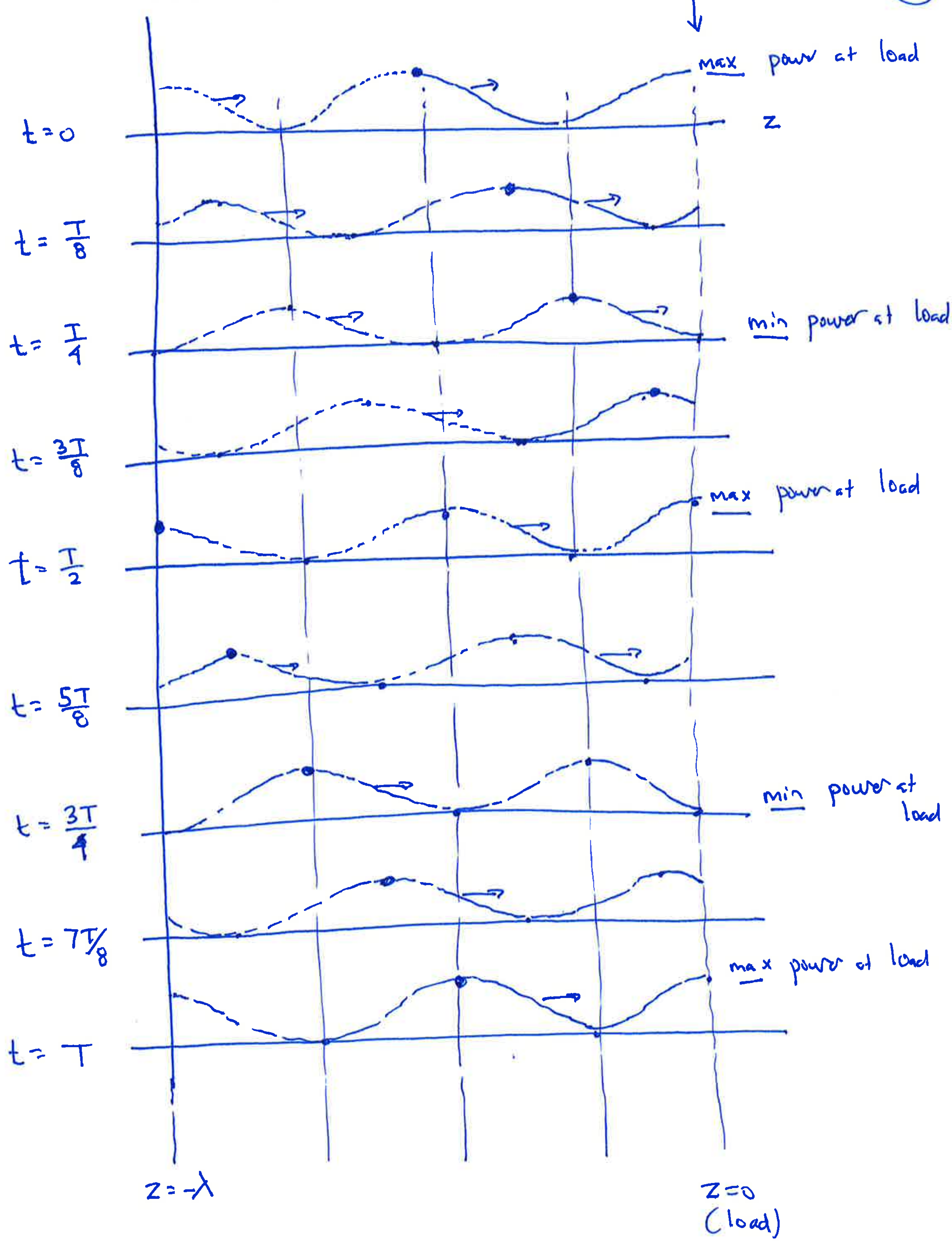
- This then becomes

$$P(z,t) = \left(V_i \cos(\omega t - kz)\right)\left(\frac{V_i}{Z_0} \cos(\omega t - kz)\right)$$

$$\boxed{P(z,t) = \left(\frac{V_i^2}{Z_0}\right) \cos^2(\omega t - kz)}$$

- What does this look like if the coax has a length of λ?

$P(z,t)$ = power along coax



- The the power delivered to the load goes through two maxima every period. The time average power is just

$$\langle P(z=0,t)\rangle = \frac{1}{T}\int_0^T P(z,t)\,dt$$

$$= \frac{1}{T}\frac{V_i^2}{Z_0}\int_0^T \cos^2(\omega t)\,dt$$

$$\langle P(z=0)\rangle = \frac{1}{2}\frac{V_i^2}{Z_0} \text{ watts}$$

- But what if $Z_L = 0$? This means that the load is essentially a short circuit. <u>No</u> voltage can develop across the load. So $V_L = V(z=0,t) = 0$

Using our equations

$$\frac{V_r}{V_i} = \frac{\cancelto{0}{Z_L} - Z_0}{\cancelto{0}{Z_L} + Z_0} = -1$$

we see that $\boxed{V_r = -V_i}$ and that

$$\frac{I_r}{I_i} = \frac{Z_0 - \cancelto{0}{Z_L}}{Z_0 + \cancelto{0}{Z_L}} = 1$$

and $\boxed{I_r = I_i}$

- This means that

$$V(z,t) = V_i \left[e^{-ikz} - e^{+ikz} \right] e^{i\omega t}$$

$$I(z,t) = I_i \left[e^{-ikz} + e^{+ikz} \right] e^{i\omega t}$$

$$V(z,t) = -2i\, V_i\, e^{i\omega t} \left[\frac{e^{ikz} - e^{-ikz}}{2i} \right]$$

$$= -2i\, V_i \left[\cos(\omega t) + i \sin(\omega t) \right] \sin(kz)$$

$$= 2V_i \sin(kz) \left[\sin(\omega t) - i\cos(\omega t) \right]$$

$$\mathrm{Re}\left[V(z,t) \right] = 2V_i \sin(kz) \sin(\omega t)$$

$$I(z,t) = 2I_i \cos(kz) \left[\cos(\omega t) + i \sin(\omega t) \right]$$

$$\mathrm{Re}\left[I(z,t) \right] = 2I_i \cos(kz) \cos(\omega t)$$

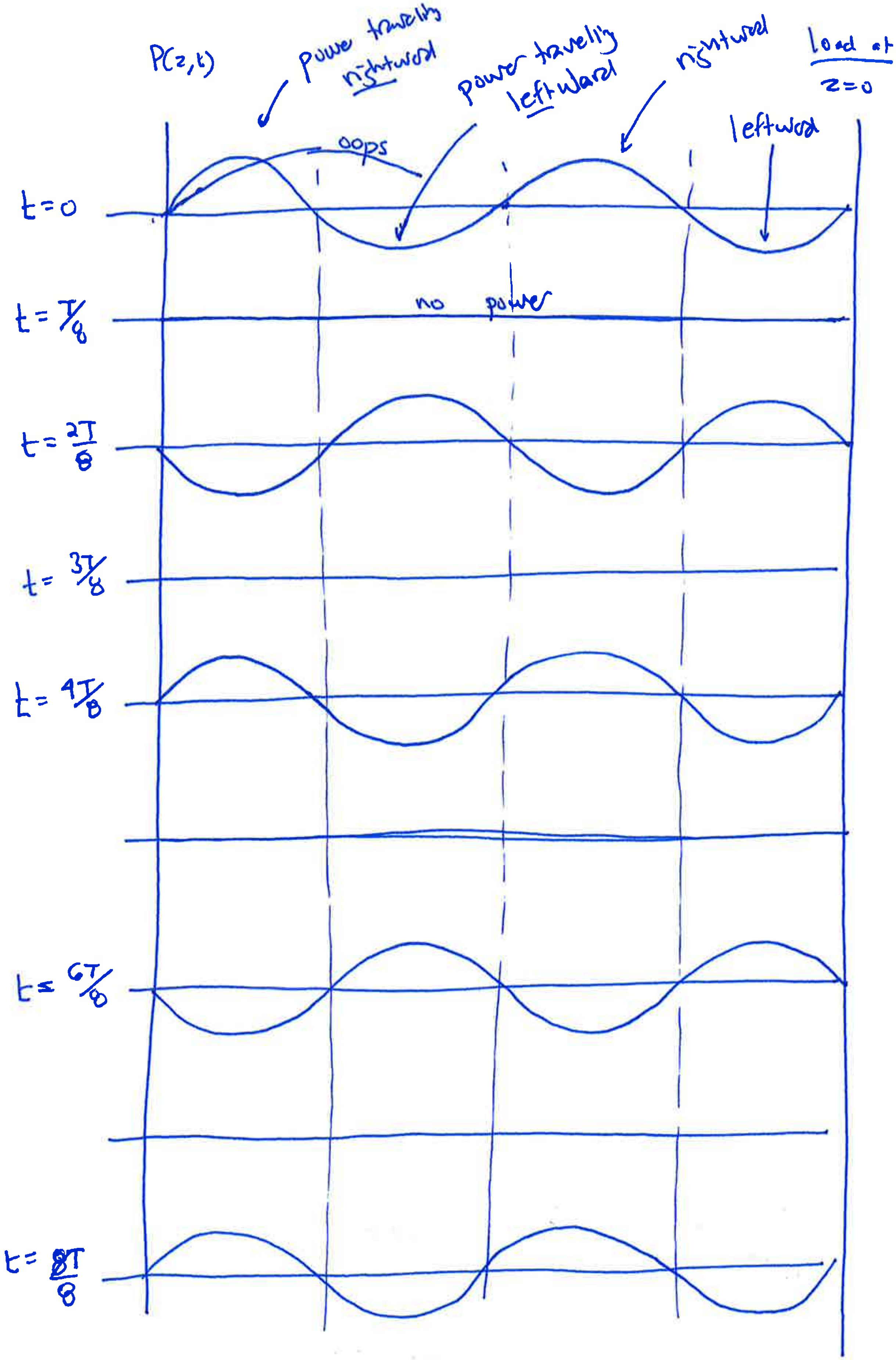
- The instantaneous power is $P(z,t) = (\mathrm{Re}\, V)(\mathrm{Re}\, I)$

$$P(z,t) = 4V_i I_i \underbrace{\sin(kz)\cos(kz)}_{\frac{1}{2}\sin(2kz)} \underbrace{\sin(\omega t)\cos(\omega t)}_{\frac{1}{2}\sin(2\omega t)}$$

$$P(z,t) = V_i I_i \sin(2kz) \sin(2\omega t)$$

$$\boxed{P(z,t) = \left(\frac{V_i^2}{Z_0} \right) \sin(2kz) \sin(2\omega t)}$$

which is a standing wave.



This is a standing wave in space and time.

There is no time average power transmission.

All incident energy is reflected.