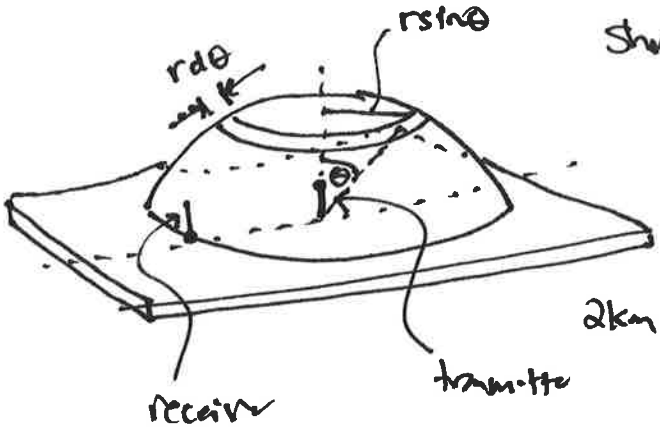
B&B 4.3



Short Antenna on surface of perfectly reflecting earth. Transmits 5kW of power at wavelength λ. What is strength of E field 2km away near earth's surface?

The total power is related to the poynting flux by integrating the poynting flux over a half-shphere (similar to eq. 4.12)

$$P(t) = \underbrace{\int_0^{\pi/2}}_{\text{half sphere}} |\bar{S}(r,t)| \underbrace{2\pi r^2 \sin\theta \, d\theta}_{dA = 2\pi r \sin\theta \, r d\theta \text{ (see above diagram)}}$$

We use $|\bar{S}| = \dfrac{q^2 a^2 \sin^2\theta}{16\pi^2 \epsilon_0 r^2 c^3}$, from $|\bar{E}| = \dfrac{-q a \sin\theta}{4\pi\epsilon_0 r c^2}$, $|\bar{B}| = \dfrac{|\bar{E}|}{c}$

Thus
$$P = \frac{q^2 a^2}{8\pi\epsilon_0 c^3} \underbrace{\int_0^{\pi/2} \sin^3\theta \, d\theta}_{\text{look up } = \frac{2}{3}} = \frac{q^2 a^2}{8\pi\epsilon_0 c^3} \frac{2}{3}$$

$$P = q^2 a^2 / 12\pi\epsilon_0 c^3$$

Now, I solve for $qa = \sqrt{P \cdot 12\pi\epsilon_0 c^3}$ and plug into $|E|$

$$|E| = \frac{-\sin\theta}{4\pi\epsilon_0 r c^2} \sqrt{12\pi\epsilon_0 c^3 P} = \frac{-\sin\theta \sqrt{3P}}{2\sqrt{\pi\epsilon_0 c}\, r}$$

And use $P = 5\text{kW}$, $\theta = 90°$, $r = 2\text{km}$ to get

$$\boxed{|E| = -0.335 \ \text{V/m}}$$