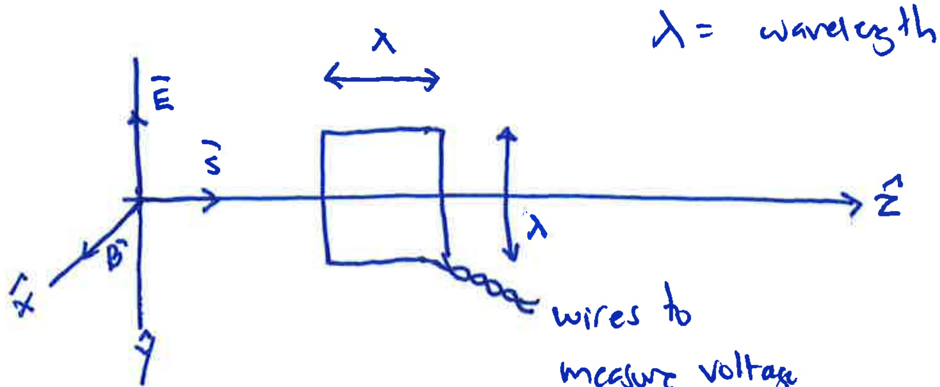
Ex 3.5

An electromagnetic wave passes by a square-loop antenna, as shown here:



The magnetic field inside the loop causes a changing flux, which (potentially) generates an electro-motive force (voltage). The electro-magnetic wave has

$$\bar{B}(x,y,z,t) = \hat{x}\, B_0 \sin(\omega t - kz)$$

a) Maxwell's eqns (specifically the ampere-maxwell law) can be used to find $E(x,y,z,t)$

$$\bar{\nabla} \times \bar{B} = \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$\nabla \times \bar{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial_x & \partial_y & \partial_z \\ B_x & \cancel{B_y}^{\,0} & \cancel{B_z}^{\,0} \end{vmatrix} = \hat{y}\left(\frac{\partial}{\partial z} B_x\right) - \hat{z}\cancel{\left(\frac{\partial}{\partial y} B_x\right)}^{\,0} =$$

$$= \hat{y}\,(-k)\, B_0 \cos(\omega t - kz)$$

Now we integrate this to get E ...

$$\bar{\nabla} \times \bar{B} = \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$-k \hat{y} B_0 \cos(\omega t - kz) = \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$-k B_0 \int \cos(\omega t - kz)\, dt = \mu_0 \epsilon_0 \int \frac{dE_y}{\partial t} dt$$

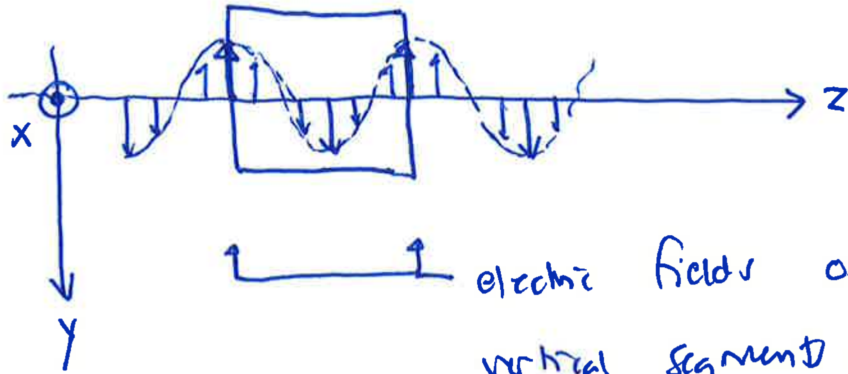
$$\frac{-k B_0}{\mu_0 \epsilon_0 \omega} \sin(\omega t - kz) = E_y$$

$$E_y = \frac{-k c^2}{\omega} B_0 \sin(\omega t - kz)$$

$$\boxed{\bar{E} = -c B_0 \sin(\omega t - kz)\, \hat{y}}$$

So when $\bar{B}$ points along $\hat{x}$ axis,
$\bar{E}$ " " $-\hat{y}$ axis.

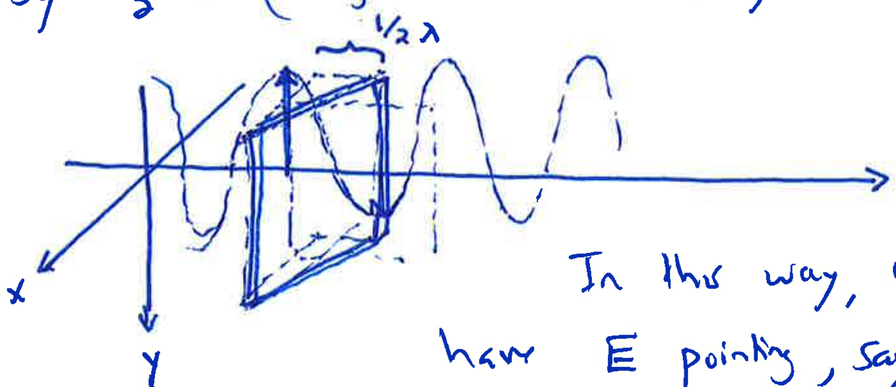
b) Is there an EMF around the loop? Actually, no. Not when the loop is oriented in the y-z plane. Why? Since the square loop has a width of λ, the electric field pointed along both vertical segments of the loop will be both pointed in the same direction ....



electric fields on both vertical segments are the same!

hence $\oint \bar{E} \cdot d\bar{s}$ = EMF around loop = 0

c) In order to maximize EMF, we would need to tip the loop out of the y-z plane so that the two vertical segments are separated by $\frac{1}{2}\lambda$ (along the z-axis), as shown here



In this way, we can have E pointing, say, up on the back vertical segment and down on the front vertical segment.

$$\text{EMF} = (\underbrace{E_{back}}_{(+\text{ number})}\lambda) - (\underbrace{E_{front}}_{(-\text{ number})}\lambda) = \boxed{2\lambda c B_0}$$