Ex 3.4 (Electro-magnetic plane wave)

The magnitude of the poynting vector $\bar{S}$ is given by

$$|\bar{S}| = S = S_0 \sin^2(x+y-\omega t)$$

This EM wave has an electric field polarized in the z-direction. Suppose x & y are measured in cm, and S_0 = constant.

a) what is λ?

$$(\bar{k}\cdot\bar{x} - \omega t) = (k_x x + k_y y + k_z z - \omega t)$$
$$= (x + y - \omega t)$$

So $k_x = 1$, $k_y = 1$, $k_z = 0$

$$k = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{2} = \frac{2\pi}{\lambda}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}\pi$$

$$\boxed{\lambda = \pi\sqrt{2}\ \text{cm}} \approx 4.4\ \text{cm}$$

b) the angular frequency $\omega = ?$

$$v = \omega/k \Rightarrow \omega = v \cdot k = c\sqrt{2}$$

$$\boxed{\omega = \sqrt{2}\,c} \approx 4.2 \times 10^{10}\ \text{cm/sec}$$

c) what is the direction of $\bar{B}$?

- How to determine the orientation of $\bar{B}$?

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

- I'll use Faraday's law.

and assume $\bar{E} = \bar{E}_0 e^{j(k_x x + k_y y + k_z z - \omega t)}$

so $\bar{\nabla} \times \bar{E} = j\bar{k} \times \bar{E}$

- Also I'll assume $\bar{B} = \bar{B}_0 e^{j(k_x x + k_y y + k_z z - \omega t)}$

so $\frac{\partial \bar{B}}{\partial t} = -j\omega \bar{B}$

$$\cancel{j}\bar{k} \times \bar{E} = \cancel{j}\omega \bar{B}$$

- I can express all these vectors in terms of their magnitudes and orientations (unit vectors)

$$k\hat{k} \times E\hat{E} = \omega B \hat{B}$$

$$\frac{kE}{\omega B}(\hat{k} \times \hat{E}) = \hat{B}$$

- Since $k/\omega = 1/c$ and $E/B = c$, we have

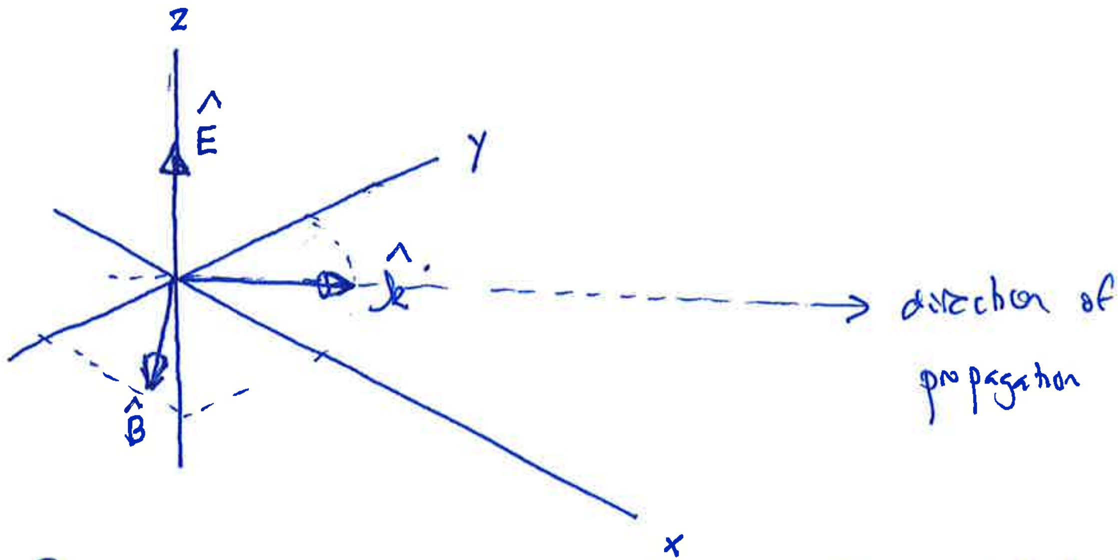
$$\hat{B} = \hat{k} \times \hat{E}$$

← This gives the orientation of $\bar{B}$.

- Since $\boxed{\hat{k} = \frac{1}{\sqrt{2}}(\hat{x} + \hat{y})}$ and $\boxed{\hat{E} = \hat{z}}$ (as we were told)

we have $\hat{B} = \frac{1}{\sqrt{2}}\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \frac{1}{\sqrt{2}}\left(\hat{x} + \hat{y}(-1) + \hat{z}(0)\right)$

or $\boxed{\hat{B} = \frac{1}{\sqrt{2}}(\hat{x} - \hat{y})}$



Here is the relationship between the orientations of $\hat{k}$, $\hat{E}$ and $\hat{B}$.

d) What are the amplitudes of $\bar{E}$ and $\bar{B}$?

Recall that the amplitude of the Poynting vector $\bar{S}$ is expressible in terms of the amplitudes of $\bar{E}$ and $\bar{B}$

$$S_0 = \frac{E_0 B_0}{\mu_0} \Rightarrow E_0 = \frac{S_0 \mu_0}{B} = \frac{S_0 \mu_0 c}{E_0}$$

$$\boxed{E_0^2 = S_0 \mu_0 c} \quad \text{also:} \quad \boxed{B_0^2 = \frac{\mu_0 S_0}{c}}$$

e) Technically, $\hat{E}$ and $\hat{B}$ can both be reversed (and they do reverse every half a period) and the wave will still be propagating in the $\hat{k}$ direction.