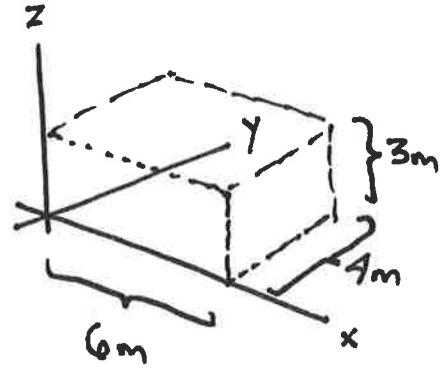
EX 2.9 (Standing acoustic waves)



- Consider a rectangular room with rigid walls, as shown here.
- I will calculate the oscillation frequencies of the lowest acoustic modes, assuming $v_{sound} \approx 330\,m/s$

- Recall (from page 147, Eq. 2.47) that at a closed/rigid wall the air velocity must be zero in the direction perpendicular to the wall. Also, the displacement, s, must be zero at the boundaries. This is because air/gas cannot vibrate -into- the wall. In 1-dimension, the general equation for the displacement of a parcel of gas was

$$s(z,t) = [A\cos(kz) + B\sin(kz)]\cos(\omega t).$$

- To enforce the boundary condition at $z=0$, we must have $s(z=0,t)=0$, which implies that $A=0$, or that $s(z,t) = B\sin(kz)\cos(\omega t)$
- Recalling EX 2.18, the pressure of the gas is

$$p = -K\left(\frac{\partial s}{\partial z}\right)$$, where $K=$ bulk modulus.

- Therefore in 1-Dimension

$$p = -KkB\cos(kz)\cos(\omega t)$$

(The pressure is a maximum at the wall, $z=0$).

- Our problem, of course, is 3-D, so our solution must be zero at $x=0$ and $y=0$ and $z=0$. So our solution must be a product of three functions that vanish (go to zero) at $x=0$, $y=0$ and $z=0$.
- A reasonable solution is

$$p(x,y,z,t) = p_0 \cos(k_x x)\cos(k_y y)\cos(k_z z)\cos(\omega t)$$

where $k_x = \frac{2\pi}{\lambda_x}$, $k_y = \frac{2\pi}{\lambda_y}$ and $k_z = \frac{2\pi}{\lambda_z}$

are the wavenumbers for standing waves in the x, y and z directions, respectively.

- Of course, the pressure must be maximum at the walls located at $x=a$, $y=b$, and $z=c$, too. This can only occur if

$$k_x a = m\pi \quad , m = 1,2,3\ldots \Rightarrow k_x = \frac{m\pi}{a}$$

$$k_y b = n\pi \quad , n = 1,2,3\ldots \Rightarrow k_y = \frac{n\pi}{b}$$

$$k_z c = q\pi \quad , q = 1,2,3\ldots \Rightarrow k_z = \frac{q\pi}{c}$$

- Our wave vector $k^2 = k_x^2 + k_y^2 + k_z^2$

or $k = \pi\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{c}\right)^2}$

- Since $\omega = vk$, this implies that our allowed oscillation frequencies are $\omega_{mnq} = \pi v\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{c}\right)^2}$

- From the last page, the allowed frequency standing waves are

$$\omega_{mnq} = \pi v \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{q}{c}\right)^2}$$

- Lets try some values for m, n, q

	m	n	q	ω_{mnq} (rad/sec)	f (Hz)
These are the lowest frequency modes.	1	0	0	172.8	27.5
	0	1	0		41.25
	0	0	1		55.0
	2	0	0		55.0
	0	1	1		68.75
	0	2	0		82.5
	0	0	2		110.0
	1	1	1		74.05
This is the next lowest mode. →	1	0	1		61.49

- Notice that the (0, 0, 1) and (2, 0, 0) are degenerate. They have the same frequency.