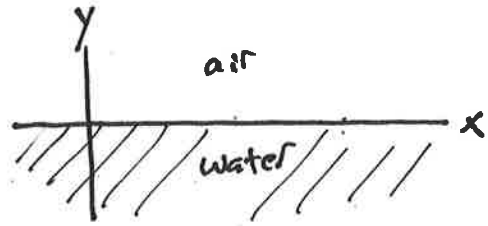
EX 2.8 (Deep water waves)

- The equation for waves in deep water (e.g. the ocean) is given by

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{2\pi}{g\lambda}\right)\frac{\partial^2 y}{\partial t^2}$$



- This looks like a wave equation with a λ-dependent-velocity.

$$\frac{\partial^2 y}{\partial t^2} = v^2(\lambda)\,\frac{\partial^2 y}{\partial x^2}\,, \quad v(\lambda) = \sqrt{\frac{g\lambda}{2\pi}}$$

What does this imply?

(a) For example, if a wave has $\lambda = 62$ meters (and $g = 9.8\,\text{m/s}^2$) then

$$v(62\text{m}) = \sqrt{\frac{(9.8)(62)}{2\pi}} = 9.8\ \text{m/s}$$

So a 62meter-long-surface wave travels at $\approx 10\,\text{m/s}$

Notice that as $\lambda\uparrow$, $v\uparrow$. This is a dispersive medium whose wave velocity depends on λ.

(b) What is $\omega(k)$, the dispersion relation?

$$v_{\text{phase}} \equiv \frac{\omega}{k} = \sqrt{\frac{g\lambda}{2\pi}} \Rightarrow \omega = k\sqrt{\frac{g}{k}}$$

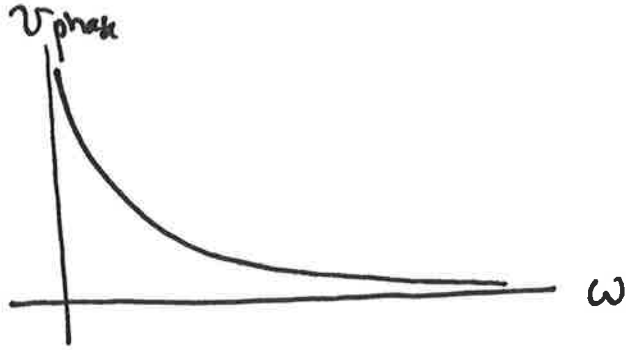
$$\boxed{\omega = \sqrt{gk}} \qquad \left(\text{since } k \equiv \frac{2\pi}{\lambda}\right)$$

or $k = \dfrac{\omega^2}{g}$

(b continued) If $\omega = \sqrt{gk}$ then

(c) The phase velocity is then, in terms of ω

$$v_{phase} = \frac{\omega}{k} = \frac{\omega}{\omega^2/g} = \frac{g}{\omega}$$



What does this imply? Since $v_{phase} \propto \sqrt{\lambda}$, very long λ waves travel with enormous phase velocities. That is why tsunami waves travel at enormous speeds (since their λ is huge). If $\lambda = 100\,km$, the $v_{phase} \approx 400\,m/s$

This is around 900 miles per hour!