BB 2.6

- Consider an isotropic spherical source of radiation.
- The intensity @ 1 meter is $5\frac{W}{m^2}$. That is $\langle S(r=1m)\rangle = 5\,\frac{W}{m^2}$
- Find $\langle S(r=100m)\rangle$, $P_o(r=100m)$, and $S_o(r=100m)$

$\langle S(r=100m)\rangle$: intensity; P_o: pressure amplitude; S_o: amplitude of vibration of air molecules

- Recall that $S = P\frac{\partial s}{\partial t}$ (instantaneous power/area or intensity)

$$= \left[\frac{A}{r}\cos(\omega t - kr)\right]\left[\frac{A}{\omega^2 \rho_o}\left\{\frac{\omega k}{r}\cos(\omega t - kr) + \frac{\omega}{r^2}\sin(\omega t - kr)\right\}\right]$$

and $\langle S\rangle = \frac{1}{T}\int_0^T S(t)\,dt$

$$= \frac{A^2\omega k}{r^2\omega^2\rho_o}\,\frac{1}{T}\int_0^T \cos^2(\omega t - kr)\,dt$$

(since other term is zero since odd × even = odd function)

$$\langle S\rangle = \frac{1}{2}\,\frac{A^2 k}{\omega\rho_o}\,\frac{1}{r^2} = \frac{1}{2}\,\frac{A^2}{\rho_o c}\,\frac{1}{r^2}$$

a) $\frac{\langle S(r=100)\rangle}{\langle S(r=1)\rangle} = \frac{1^2}{100^2}$ (since both go as $1/r^2$)

so $\langle S(r=100)\rangle = \frac{1}{100^2}\langle S(r=1)\rangle = \boxed{5\times10^{-4}\,\frac{W}{m^2}}$.

b) since $\langle S(r=1)\rangle = 5 = \frac{1}{2}\frac{A^2}{\rho_o c}\frac{1}{1^2}$

then $A^2 = 10\rho_o c$ and $A = \sqrt{10\rho_o c}$

using $\rho_o = 1.29$, $c = 331$ $\Rightarrow$ $A \approx 65$

Since $P = P_o\cos(\omega t - kr)$ and $P_o = \frac{A}{R}$ then $P_o = \frac{65}{100}$

at $r = 100$ meters. So $\boxed{P_o = 0.65 \text{ Pascals}}$

c) Finally, since

$$S_o(r) = \frac{A}{\omega^2 \rho_o} \frac{\ell}{r}$$

$$= \frac{\sqrt{10\rho_o c}}{\omega c \rho_o r}$$

$$S_o(100\text{ meters}) = \sqrt{\frac{10}{\rho_o c}} \frac{1}{100\omega} \approx \boxed{7.7\ \mu m}$$

The amplitude of motion of the air molecules is just under 8 microns.