

Ex 2.5

Pressure oscillations in an open pipe



General solution:  $p(z, t) = [A \cos(kz) + B \sin(kz)] \cos(\omega t)$

Boundary conditions:  $p(z=0) = p(z=L) = 0$  (i & ii)

$p(z=L/2) = p_0$  @  $t=0$  (iii)

Apply B.C. (i):  $0 = [A \cos(0) + B \sin(0)] \cos(\omega t)$

$0 = A \cos(\omega t) \Rightarrow \boxed{A=0}$

Apply B.C. (iii)  $p_0 = B \sin(k \frac{L}{2}) \cos(\omega t)$

$p_0 = B \sin(k \frac{L}{2}) \Rightarrow \boxed{B=p_0}$

Apply B.C. (ii)  $0 = p_0 \sin(k \frac{L}{1}) \cos(\omega t)$

$\Rightarrow kL = n\pi \Rightarrow \boxed{k_n = \frac{n\pi}{L}}$

Since  $v = \frac{\omega}{k} \Rightarrow \omega_n = v k_n$

$\boxed{\omega_n = \frac{n\pi v}{L}}$

where  $v$  = speed of sound.