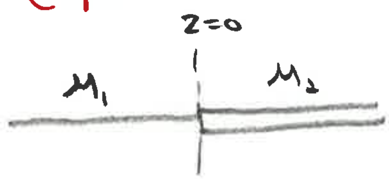
BB 2.3 (4pts)



$v = \frac{\omega}{k} = \sqrt{\frac{T}{\mu}}$

$f_I = A_I e^{i(\omega t - k_I z)}$ $\quad z<0$

$f_R = A_R e^{i(\omega t + k_R z)}$ $\quad z<0$

$f_T = A_T e^{i(\omega t - k_T z)}$ $\quad z>0$

$$f = f_I + f_R = A_I e^{i(\omega t - k_I z)} + A_R e^{i(\omega t + k_R z)} \qquad z<0$$
$$= A_T e^{i(\omega t - k_T z)} \qquad z>0$$

- At boundary

  i) $f(0^-,t) = f(0^+,t)$ continous

  ii) $\left.\frac{\partial f}{\partial z}\right|_{0^-} = \left.\frac{\partial f}{\partial z}\right|_{0^+}$ continuous derivative

- So i) $A_I + A_R = A_T$

  ii) $-k_I A_I + k_R A_R = -k_T A_T$

- But $k = \frac{\omega}{v} = \omega\sqrt{\frac{\mu}{T}}$ and $\frac{kT}{\omega} = \frac{\omega\sqrt{\mu T}}{\omega} = Z$

  ii) $+Z_1 A_I - Z_1 A_R = +Z_2 A_T$

- And $A_R = A_T - A_I$

$$= \cancel{A}\frac{Z_1(A_I - A_R)}{Z_2} - A_I = \frac{Z_1}{Z_2}A_I - \frac{Z_1}{Z_2}A_R - A_I$$

$$Z_2 A_R\left(1 + \frac{Z_1}{Z_2}\right) = A_I\left(\frac{Z_1}{Z_2} - 1\right) Z_2$$

$$A_R(Z_2 + Z_1) = A_I(Z_1 - Z_2)$$

$$\boxed{A_R = A_I\left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)}$$

$$A_T = A_R + A_I$$

$$= A_I\left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right) + A_I\left(\frac{Z_1 + Z_2}{Z_1 + Z_2}\right)$$

$$\boxed{A_T = A_I\left(\frac{2Z_1}{Z_1 + Z_2}\right)}$$

- Finally

$$\frac{A_R}{A_T} = \frac{Z_1 - Z_2}{\cancel{Z_1 + Z_2}} \quad \frac{\cancel{Z_1 + Z_2}}{2Z_1}$$

$$= \frac{Z_1 - Z_2}{2Z_1}$$

$$\boxed{R = \frac{Z_{01} - Z_{02}}{2Z_{01}}}$$

$$\boxed{\frac{A_R}{A_I} = \frac{Z_1 - Z_2}{Z_1 + Z_2}}$$