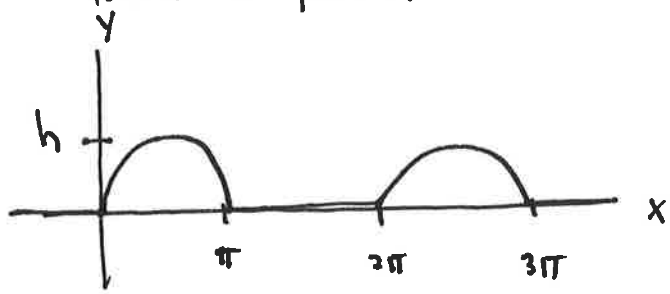
Ex 2.10 (Half wave rectifier fourier series)

- We wish to express the following function in terms of its fourier components



$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ h\sin(x) & 0 \le x \le \pi \end{cases}$$

$$f(x) = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos(mx) + \sum_{m=1}^{\infty} B_m \cos(mx)$$

- First, find A_0

$$A_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} h\sin(x)\,dx\right]$$

$$= \frac{1}{\pi}\left[0 + -h\cos(x)\Big|_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-h(-1) + h(1)\right]$$

$$\boxed{A_0 = 2h/\pi}$$

- Now find A_m

$$A_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(mx)\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\cos(mx)\,dx + \frac{h}{\pi}\int_{0}^{\pi} \sin(x)\cos(mx)\,dx$$

wolfram alpha

$$= \boxed{\frac{h}{\pi}\,\frac{\cos(m\pi)+1}{(1+m)(1-m)} = A_m}$$

- Now B_m

$$B_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(mx)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} 0\sin(mx)\,dx + \frac{h}{\pi}\int_{0}^{\pi}\sin(x)\sin(mx)\,dx$$

$$\boxed{B_m = \frac{h}{\pi}\,\frac{-\sin(m\pi)}{m^2-1}}$$

- Notice that $A_m = 0$ for odd m. And $B_m = 0$ if $m \neq 1$.

For $m=1$, we need to use L'Hopital's rule to find B_1

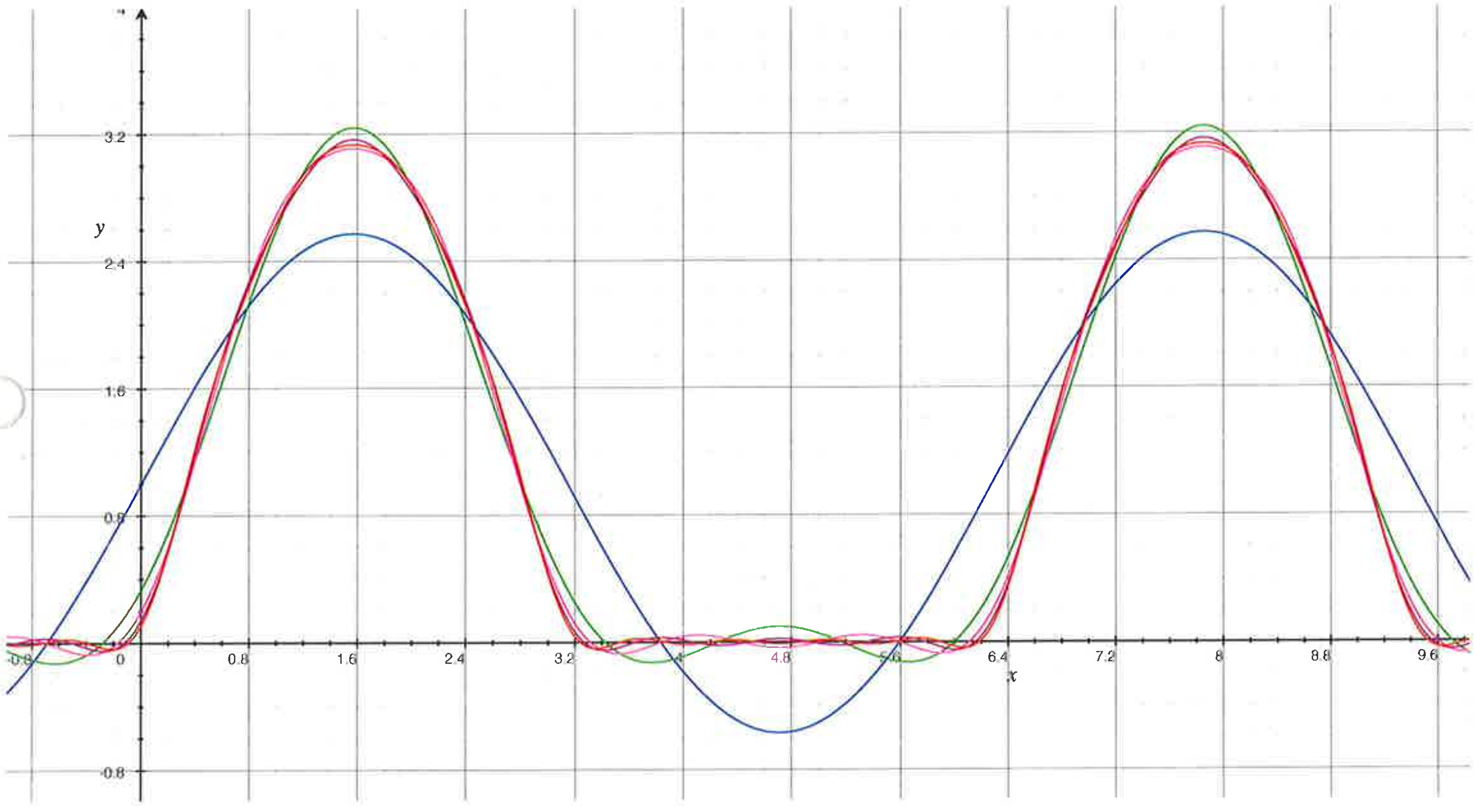
$$B_1 = \frac{h}{\pi}\,\frac{-\pi\cos(m\pi)}{2m} = \frac{h}{\pi}\,\frac{\pi}{2} = \frac{h}{2}$$

- Let's find our values of A_m & B_m by plugging in....

m	A_m	B_m
1	0	$h/2$
2	$\frac{h}{\pi}\left(\frac{-2}{1\cdot 3}\right)$	0
3	0	0
4	$\frac{h}{\pi}\left(\frac{-2}{3\cdot 5}\right)$	0
5	0	0
6	$\frac{h}{\pi}\left(\frac{-2}{5\cdot 7}\right)$	0

• Finally, our Fourier series is

$$f(x) = \frac{h}{\pi}\left(1 + \frac{\pi}{1\cdot 2}\sin(x) - \frac{2}{1\cdot 3}\cos(2x) - \frac{2}{3\cdot 5}\cos(4x) - \frac{2}{5\cdot 7}\cos(6x) - \frac{2}{7\cdot 9}\cos(8x) - \dots\right)$$



Blue	2 terms
Green	3 terms
Orange	4 terms
Purple	5 terms
Red	6 terms

As I include more terms, the approximation to a rectified sine wave gets better and better.