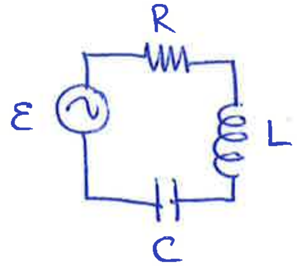
BB 1.9



$$\mathcal{E} = \mathcal{E}_0 \cos\omega t$$

$$\mathcal{E} = IR + L\frac{dI}{dt} + \frac{Q}{C}$$

$$\frac{d^2Q}{dt^2} + \beta\frac{dQ}{dt} + \omega_0^2 Q = \frac{\mathcal{E}}{L} \quad , \beta = \frac{R}{L}, \quad \omega_0^2 = \frac{1}{LC}$$

$$Q = Q_0 e^{j(\omega t+\delta)}$$

$$\left(-\omega^2 + \beta j\omega + \omega_0^2\right) Q_0 e^{j(\omega t+\delta)} = \frac{\mathcal{E}_0}{L} e^{j\omega t}$$

$$\left(\omega_0^2 - \omega^2 + \beta j\omega\right) Q_0 = \frac{\mathcal{E}_0}{L} e^{-j\delta}$$

$$\left(\omega_0^2 - \omega^2\right)\frac{Q_0 L}{\mathcal{E}_0} = \cos\delta$$

$$-\beta\omega\frac{Q_0 L}{\mathcal{E}_0} = \sin\delta$$

$$\tan\delta = \frac{-\beta\omega}{\omega_0^2 - \omega^2}$$

$$\cancel{Q_0}\quad \left(\omega_0^2-\omega^2\right)^2\left(\frac{Q_0 L}{\mathcal{E}_0}\right)^2 + (\beta\omega)^2\left(\frac{Q_0 L}{\mathcal{E}_0}\right)^2 = 1$$

$$Q_0 = \frac{\mathcal{E}_0/L}{\sqrt{\left(\omega_0^2-\omega^2\right)^2 + (\omega\beta)^2}} \quad , \quad Q = Q_0 e^{j(\omega t+\delta)}$$

$$V_c = \frac{Q}{C}$$

$$V_{co} = \frac{Q_0}{C}$$

$$\frac{dV_{co}}{d\omega} \equiv 0 = 2\left(\omega_0^2-\omega^2\right)(-2\omega) + 2\omega\beta^2$$

$$0 = 2(\omega_0^2-\omega^2)(-2\omega)+2\omega\beta^2$$

$$\beta^2-2(\omega_0^2-\omega^2)=0$$

$$\omega^2 = \omega_0^2 - \beta^2/2$$

$$\omega = \sqrt{\omega_0^2-\beta^2/2}$$

$$\boxed{\omega = \sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}}$$

b) $I = \frac{dQ}{dt} = +j\omega Q$

$$\frac{dI}{dt} = -\omega^2 Q$$

$V_L = L\frac{dI}{dt}$, $\frac{dV_L}{d\omega}=0$ when $\frac{d}{d\omega}(\omega^2 Q_0)=0$

$$2\omega Q_0 + \omega^2\frac{dQ_0}{d\omega}=0$$

$$\frac{2\omega}{\sqrt{(\omega^2-\omega_0^2)^2+(\beta\omega)^2}} + \omega^2\left(-\frac{1}{\cancel{2}}\right)\frac{\cancel{2}(\omega^2-\omega_0^2)(2\omega)+\cancel{2}\beta^2\omega}{\left((\omega^2-\omega_0^2)+(\beta\omega)^2\right)^{3/2}}=0$$

$$\frac{2\cancel{\omega}}{2} - \frac{\omega^2}{2}\cdot\frac{2\cancel{\omega}(\omega^2-\omega_0^2)+\beta^2\cancel{\omega}}{(\omega^2-\omega_0^2)^2+(\beta\omega)^2}=0$$

$$\frac{\omega^2(\omega^2-\omega_0^2)+\beta^2\omega^2/2}{(\omega^2-\omega_0^2)^2+(\beta\omega)^2}=1$$

$$\cancel{\omega^4}-\omega_0^2\omega^2+\frac{\beta^2\omega^2}{2} = \cancel{\omega^4}+\omega_0^4-2\omega^2\omega_0^2+\beta^2\omega^2$$

$$\omega^2\omega_0^2 = \omega_0^4+\beta^2\omega^2/2$$

$$\omega^2(\omega_0^2-\beta^2/2)=\omega_0^4$$

$$\omega^2 = \frac{\omega_0^4}{\omega_0^2-\beta^2/2} = \frac{1}{\frac{1}{\omega_0^2}-\frac{\beta^2}{2\omega_0^4}}$$

$$\boxed{\frac{1}{\omega} = \sqrt{LC-\frac{R^2C^2}{2}}}$$