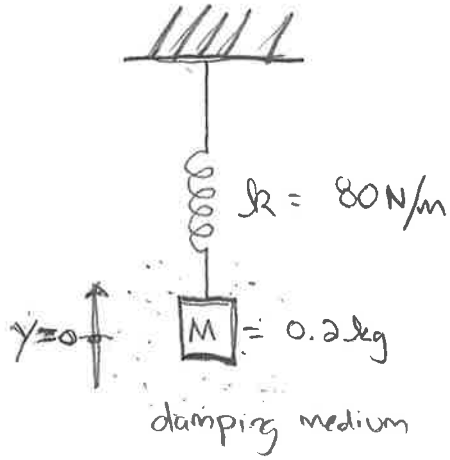
BB 1.7



$$\underbrace{}_{}\text{spring} \quad \text{damp} \quad \text{grav.}$$

$$m\ddot{x} = -kx - b\dot{x} - mg$$

$$\ddot{x} + \frac{k}{m}x + \frac{b}{m}\dot{x} + g = 0$$

- This is inhomogeneous, but let's redefine $y \equiv x + \frac{mg}{k}$ so that it becomes homogeneous.

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = 0$$

a) Our differential equation is now

$$\boxed{\ddot{y} + \beta\dot{y} + \omega_0^2 y = 0}, \quad \beta \equiv \frac{b}{m}, \quad \omega_0^2 = \frac{k}{m}$$

b) when pulled down & released, $\omega = \frac{\sqrt{3}}{2}\omega_0$. what does this imply about Q? Since it is oscillating, let's presume it is lightly damped. So, by (Eq. 1.74)

$$\omega_1^2 = \omega_0^2 - \beta^2/4$$

$$\beta^2 = 4\omega_0^2 - \omega_1^2$$

$$\beta = \sqrt{4\omega_0^2 - \left(\frac{\sqrt{3}}{2}\omega_0\right)^2} \quad \leftarrow \text{using } \omega_1 = \frac{\sqrt{3}}{2}\omega_0$$

$$\beta = 2\omega_0\sqrt{1 - 3/4} = \omega_0$$

Now what is Q? By eqn. (1.109)

$$Q = \frac{\omega^2 + \omega_0^2}{2\beta\omega} \quad \text{Plugging in gives}$$

$$\boxed{Q = \frac{1 + 3/4}{\sqrt{3}} = 1.01}$$

c) If the system is critically damped, $\frac{b}{2m} = \sqrt{\frac{k}{m}}$.

Show that $x = e^{-bt/2m}(A+Bt)$ is a solution.

Here goes:

$$\dot{x} = \frac{-b}{2m} e^{-bt/2m} (A+Bt) + Be^{-bt/2m}$$

$$\ddot{x} = \frac{b^2}{4m^2} e^{-bt/2m} (A+Bt) - \frac{b}{2m} e^{-bt/2m} B - \frac{b}{2m} e^{-bt/2m} B$$

$$e^{-bt/2m} \left\{ \frac{b^2}{4m^2}(A+Bt) - \frac{b}{m} B \right\} + Be^{-bt/2m} \left\{ \frac{-b}{2m} (A+Bt) + B \right\}$$

$$+ \omega_0^2 e^{-bt/2m} \{A + Bt\} \overset{?}{=} 0$$

$$\beta = \frac{b}{m}, \quad \omega_0^2 = \frac{k}{m} = \left(\frac{b}{2m}\right)^2 = \frac{1}{4}\beta^2$$

$$\frac{1}{4}\beta^2 \cancel{(A+Bt)} - \beta B \} + \{ -\frac{1}{2}\beta^2 \cancel{(A+Bt)} + B\beta \} + \{ \frac{1}{4}\beta^2 \cancel{(A+Bt)} \} = 0$$

Checks

d) If $v(t=0) = 3$ and $y(t=0) = 0$,

find time of maximum displacement, t^*.

$$y(t) = (A + Bt)e^{-bt/2m}$$

$$y(t=0) = 0 = A = 0$$

So $y(t) = Bte^{-bt/2m}$

$$\dot{y}(t) = B\left(1 - \frac{\beta}{2}t\right)e^{-\beta t/2}$$

$$\dot{y}(t=0) = B = 3$$

$$\dot{y}(t^*) = 0 = B\left(1 - \frac{\beta}{2}t^*\right)e^{-\beta t^*/2}$$ At max. displacement.

So $t^* = \frac{2}{\beta} = \frac{2m}{b} = \sqrt{\frac{m}{k}}$

$$t^* = \sqrt{\frac{0.2}{80}} = \boxed{0.05\text{sec}}$$

And $y(t^*) = (e^{-1})(3)(0.05) = \boxed{5.52 \text{ cm}}$