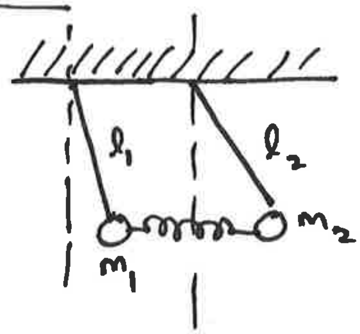
EX 1.15



$m_1 = m_2$

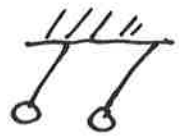
$l = 0.4\,m = l_1 = l_2$

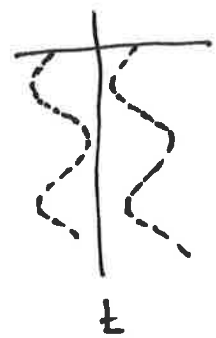
$g = 9.8\,m/s^2$ -

$T_{1\ \text{pendulum}} = 1.25\,sec$

- Thankfully, we can use our solution to EX 1.14 to obtain the "normal modes" of oscillation.

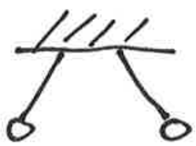
- Symmetric mode: (lower frequency)

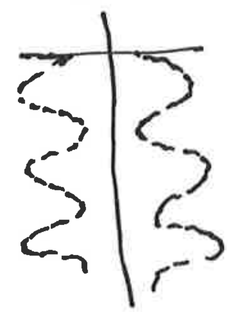




$$\omega_S = \sqrt{\frac{g}{l}}$$

- Anti-symmetric mode (higher frequency)

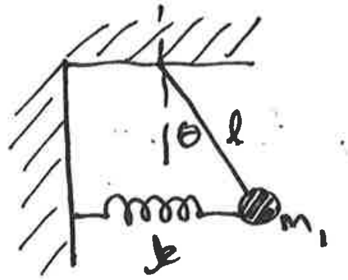




$$\omega_A = \sqrt{\frac{g}{l} + \frac{2k}{M}}$$

- We can find the spring constant k from the fact that, when one pendulum is clamped, the other oscillates with frequency $\omega_1 = \frac{2\pi}{T_1} = 5.027\ \frac{rad}{sec}$

- For small oscillations of 1 pendulum attached to a spring, what is the frequency? Is it just $\omega = \sqrt{k/m}$? Probably not....



The equation of motion (from Ex 1.14) is

$$\ddot{x}_1 = -x_1\left(\frac{k}{m} + \frac{g}{l}\right)$$

So the frequency is $\omega_1 = \sqrt{\frac{k}{m} + \frac{g}{l}}$

Plug in our numbers:

$$\frac{2\pi}{T_1} = \sqrt{\left(\frac{k}{m}\right) + \frac{g}{l}}$$

$$\left(\frac{2\pi}{T_1}\right)^2 - \frac{g}{l} = \left(\frac{k}{m}\right)$$

$$\left(\frac{k}{m}\right) = \left(\frac{2\pi}{1.25}\right)^2 - \frac{9.8}{0.4} =$$

$$\left(\frac{k}{m}\right) = 0.766$$

(a) I can use this now to find ω_S & ω_A

$$\omega_S = \sqrt{\frac{9.8}{0.4}} = 4.95\ \text{rad/sec} = \boxed{0.788\ \text{Hz}}$$

$$\omega_A = \sqrt{\frac{9.8}{0.4} + 2(0.766)} = 5.1\ \frac{\text{rad}}{\text{sec}} = \boxed{0.812\ \text{Hz}}$$

(b) If we do not excite one of the normal modes, then the oscillations are more complicated. First, one pendulum oscillates with a large amplitude, then the other, then back to the first. What is the time interval between max amplitudes of one of the pendulums? We can use (eq. 1.180) or eq (1.181)

$$X_1 = X_{10} \cos\Big(\underbrace{\frac{\omega_A - \omega_S}{2}}_{\substack{\text{low frequency}\\ \text{oscillation of}\\ \text{amplitude}}} t\Big) \cos\Big(\underbrace{\frac{\omega_A + \omega_S}{2}}_{\substack{\text{high frequency}\\ \text{oscillation}}} t\Big)$$

- In a time period T given by

$$\left(\frac{\omega_A - \omega_S}{2} T\right) = 2\pi$$

the amplitude of one pendulum will go through an entire cycle. So

$$T = \frac{2\pi \cdot 2}{(\omega_A - \omega_S)} = \frac{4\pi}{(5.1 - 4.95)}$$

$$\boxed{T = 83.8 \text{ seconds}}$$