CLASSICAL MECHANICS

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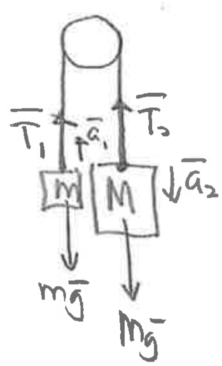
Homework 4

First-pass presentations on Friday, Sept. 25, 2020.

Full written solutions due Monday, Sept. 28, 2020.

(1) **Atwood machine** An atwood machine consists of a pulley (of negligible mass) over which two blocks, of mass m and M, are hung by a cord. Find the tension in the cord and the acceleration of the larger mass, M.



$$\left.\begin{aligned} m a_1 &= T_1 - mg \\ m a_2 &= Mg - T_2 \end{aligned}\right\} \text{4 unknowns } a_1, a_2, T_1, T_2$$

$T_1 = T_2 = T$ if pulley is massless (doesn't take force to rotate)

$a_1 = a_2 = a$ if string doesn't stretch

$$\left.\begin{aligned} m a &= T - mg \\ M a &= Mg - T \end{aligned}\right\} \text{2 unknowns } a, T$$

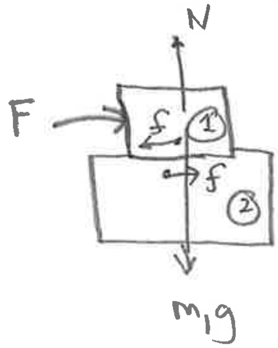
eliminate a $\Longrightarrow$ $\dfrac{T - mg}{m} = \dfrac{Mg - T}{M}$ $\Longrightarrow$ $\boxed{T = 2g\dfrac{mM}{m+M}}$

find a $\Longrightarrow$ $a = \dfrac{1}{m}\left[2g\dfrac{mM}{m+M} - mg\right]$

$$\boxed{a = g\left(\frac{M-m}{M+m}\right)}$$

(2) **Pushed blocks** A block of mass M_1 rests atop a block of mass M_2, which lies on a frictionless table. The coefficient of friction between the blocks is μ. What is the

maximum horizontal force that can be applied to the blocks for them to accelerate without slipping on one another if the force is applied to block 1? To block 2?



Block 1: $m_1 a = F - f$

2: $m_2 a = f$

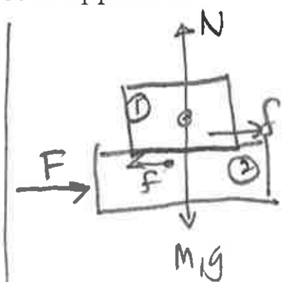
But $f \le \mu N = \mu m_1 g$

eliminate a:

$$\frac{F-f}{m_1} = \frac{f}{m_2}$$

$$F = f\left(\frac{m_1}{m_2} + 1\right)$$

$$\boxed{F \le \mu m_1 g \left(\frac{m_1}{m_2} + 1\right)}$$



1 $m_1 a = f$

2 $m_2 a = F - f$

eliminate a:

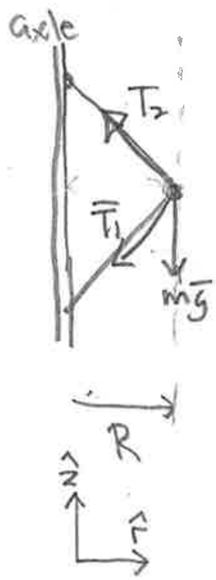
$$\frac{f}{m_1} = \frac{F-f}{m_2}$$

$$F = f\left(\frac{m_2}{m_1} + 1\right)$$

$$F \le \mu m_1 g \left(\frac{m_2}{m_1} + 1\right)$$

$$\boxed{F \le \mu g (m_2 + m_1)}$$

(3) **Ball and axle.** A mass m is connected to a vertical revolving axle by two strings, each of length l. While swinging around at angular speed ω, the strings form a 45-45-90 triangle with the ball at the 90 degree angle and the sides of length l. Gravity is directed downward, of course. Draw a clear force diagram for m, and then find the tension in each string.



$$\bar{a} = -\omega^2 R \hat{r}$$

$$R = \frac{1}{\sqrt{2}} l$$

$$\bar{T}_1 = T_1(-\hat{r}\sin 45^\circ - \hat{z}\cos 45^\circ)$$

$$= \frac{1}{\sqrt{2}} T_1(-\hat{r} - \hat{z})$$

$$\bar{T}_2 = T_2\left(-\hat{r}\frac{1}{\sqrt{2}} + \hat{z}\frac{1}{\sqrt{2}}\right)$$

$$m\bar{g} = -mg\hat{z}$$

$$\bar{F} = m\bar{a}$$

$\hat{z}$ component:

$$0 = \frac{T_2}{\sqrt{2}} - \frac{T_1}{\sqrt{2}} - mg$$

$\hat{r}$ component:

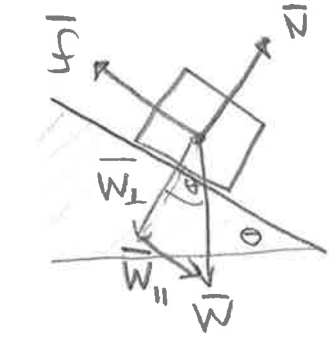
$$-m\omega^2 R = -\frac{T_1}{\sqrt{2}} - \frac{T_2}{\sqrt{2}}$$

2 eqns 2 unknowns

$$\boxed{T_1 = \frac{m}{2}\left(\omega^2 l - \sqrt{2} g\right) \qquad T_2 = \frac{m}{2}\left(\omega^2 l + \sqrt{2} g\right)}$$

(4) **Block and wedge.** A block rests on a wedge inclined at angle θ. The coefficient of friction between the block and plane is μ. First, find the maximum value of θ for the block to remain motionless on the wedge when the wedge is fixed in position. Next, if the wedge is given a horizontal acceleration a, and assuming that $\tan\theta < \mu$, find the minimum acceleration for the block to remain on the wedge without sliding. Finally, repeat the previous part, but find the maximum value of the acceleration.

a)



– perpendicular forces:

$$N - W_\perp = m a_\perp$$

$$N - mg\cos\theta = 0$$ ← stays on plane

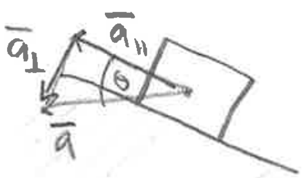
– parallel force:

$$W_{\parallel} - f = m a_{\parallel}$$

$$mg\sin\theta - \mu N = 0$$ ← to stick

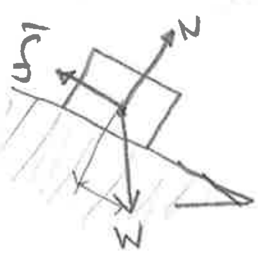
Combine to get: $\boxed{\tan\theta \le \mu}$ for sticking (not sliding)

b)



Decompose acceleration into parts $\perp$ and $\parallel$ to plane

Use $F = ma$



$$m a_{\parallel} = f - mg\sin\theta \quad \text{} \} \text{ parallel}$$

$$m a_\perp = -N + mg\cos\theta \quad \} \text{ perpendicular}$$

$$\{ f \le \mu N, \; a_{\parallel} = a\cos\theta, \; a_\perp = a\sin\theta \} \text{ other eqns.}$$

2 eqns, 2 unknowns a and N

$$\begin{cases} ma\cos\theta \le \mu N - mg\sin\theta \\ ma\sin\theta = -N + mg\cos\theta \end{cases}$$ ← solve for N and plug into 1st eqn

$$a(\cos\theta + \mu\sin\theta) \le \mu g\cos\theta - g\sin\theta$$

$$a \le g\left(\frac{\mu\cos\theta - \sin\theta}{\cos\theta + \mu\sin\theta}\right) \quad \text{or} \quad \boxed{a \le g\left(\frac{\mu - \tan\theta}{1 + \mu\tan\theta}\right)}$$

This is max accel in negative direction