CLASSICAL MECHANICS

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Homework 3

First-pass presentations on Friday, Sept. 18, 2020.

Full written solutions due Monday, Sept. 21, 2020.

(1) **Mass and force.** A 5 kg mass moves under the influence of a force $\vec{F} = (4t^2\hat{\imath} - 3t\hat{\jmath})$ newtons, where t is the time in seconds. It starts from the origin at $t = 0$. Find (a) its velocity; (b) its position; and (c) $\vec{r} \times \vec{v}$ for any later time.

a) $\vec{v} - \cancel{\vec{v}_0}^{\,0} = \frac{1}{m} \int_0^t (4t^2\hat{\imath} - 3t\hat{\jmath})\, dt$

$\vec{v} = \frac{1}{m}\left(\frac{4}{3}t^3\hat{\imath} - \frac{3}{2}t^2\hat{\jmath}\right) = \frac{4}{15}t^3\hat{\imath} - \frac{3}{10}t^2\hat{\jmath}$

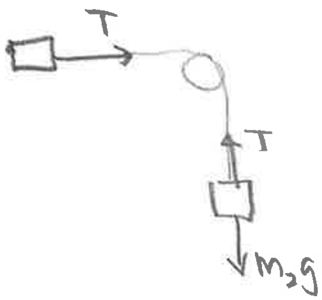
b) $\vec{r} - \cancel{\vec{r}_0}^{\,0} = \int_0^t \vec{v}\, dt = \frac{1}{15}t^4\hat{\imath} - \frac{1}{10}t^3\hat{\jmath}$

c) $\vec{r} \times \vec{v} = \left(\frac{t^4}{15}\right)\left(-\frac{3}{10}t^2\right)(\hat{\imath} \times \hat{\jmath}) + \left(-\frac{t^3}{10}\right)\left(\frac{4t^3}{15}\right)(\hat{\jmath} \times \hat{\imath})$

$= \frac{1}{150}t^6\hat{k}$

(2) **Blocks connected by pulley.** Mass M_1 lies on the surface of a frictionless table. The string runs over a pulley, and is connected to another mass, M_2, which is

suspended from the string next to the table. If the system is released from rest, find how far block M_1 slides in time t.



① $m_1 a_1 = T$

② $m_2 a_2 = m_2 g - T$

2 eqns, 3 unknowns

③ $a_1 = a_2$ so rope doesn't stretch

$m_1 a_1 + m_2 a_1 = m_2 g$

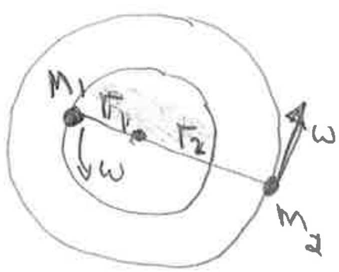
$\rightarrow a_1 = g\left(\frac{m_2}{m_1+m_2}\right)$

$v_1 = g\left(\frac{m_2}{m_1+m_2}\right)t$

$x_1 = \frac{1}{2} g\left(\frac{m_2}{m_1+m_2}\right)t^2$

(3) **Orbiting masses.** Two particles of mass m and M undergo uniform circular motion about each other at a separation R under the influence of an attractive force F. The angular velocity is ω radians per second. Show that $R = (\frac{F}{\omega^2})(\frac{1}{m} + \frac{1}{M})$

N3L $\Rightarrow \overline{F}_{12} = -\overline{F}_{21} \Rightarrow m a_2 = m a_1 = F$



a_1 & a_2 are centripetal accel.

so $a_1 = \omega^2 r_1$, $a_2 = \omega^2 r_2$

so $r_1 = \frac{a_1}{\omega_1^2} = \frac{F}{m_1 \omega^2}$

$r_2 = \frac{F}{m_2 \omega^2}$

Since $R = r_1 + r_2 \Rightarrow R = \frac{F}{\omega^2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right)$