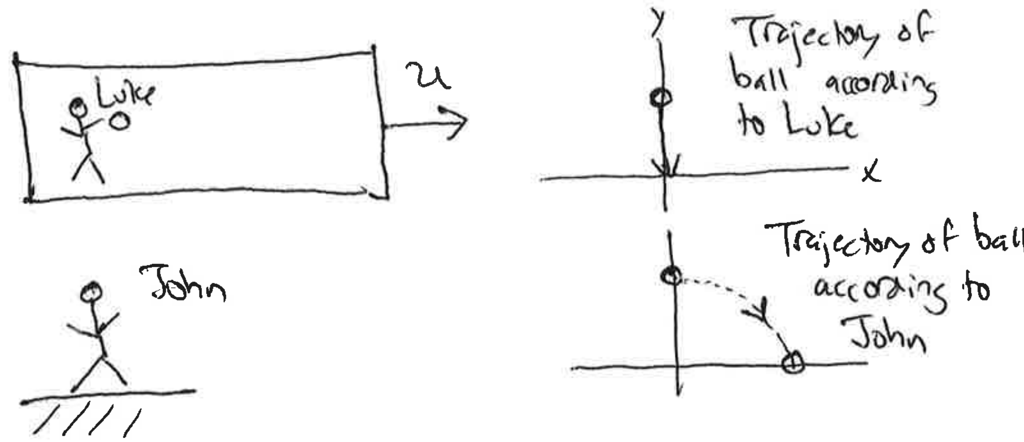
Lecture on special relativity — K. Kuehn

- Einstein published his <u>special</u> theory of relativity in 1905 and his <u>general</u> theory of relativity in 1916. We will focus on his special theory, but make mention of his general theory on a number of occasions as opportunity arises.

- Things appear differently to different observers.  For example, a table-top may appear to be a square from above  but a thin rectangle from the side . This non-controversial point was made by Descartes in the 17th century ~~by French philosopher René Descartes~~ in his <u>Meditations on First Philosophy</u>. Which observation is correct? Both.

- This is also the case for reports of events. For example, the <u>scene of a crime</u> may be reported differently by different witnesses who assess the events relating to the crime from different perspectives. They can say different things without necessarily contradicting one another.

- These considerations apply also to the reported trajectory of a ball dropped on a train.



- Luke, on the train, reports a linear trajectory of the dropped ball. John, watching the train race past at speed u, reports a parabolic trajectory of the dropped ball.
- Which observer sees the true trajectory? Both. They are simply in different reference frames which are moving relative to one another at speed u.
- Notice that the ball obeys Newton's laws of motion according to both observers, Luke & John.

- The Principle of Relativity states that Newton's laws of motion – and any other laws of physics – must be obeyed when studied by any observer moving at a constant velocity.

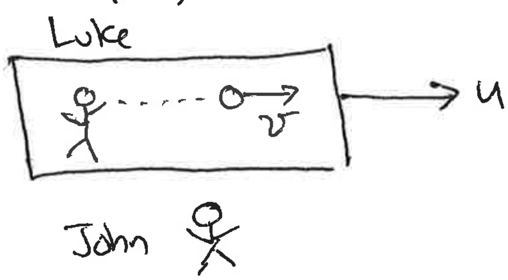
- Stated differently: you cannot tell if you are at rest or in motion at constant velocity by performing any experiment.

- Any such observer – who is not accelerating – is called an inertial observer, and any coordinate system attached to such an observer is called an inertial coordinate system (also, a Galilean system of coordinates).

- So: a person travelling at constant velocity on a space ship is an inertial observer, while a person in a car rounding a corner is not an inertial observer.

- Now let me introduce a <u>paradox</u> that arises when considering two inertial observers who are not in the same inertial reference frame

- Luke, riding in a spaceship at speed u relative to John, an astronaut outside the ship, throws a baseball at speed v.

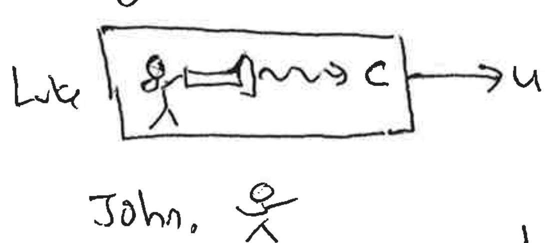


According to Luke, the baseball speed is u

According to John, " " " " $V = v + u$

- In other words, you can just add the speed of the ship to the speed of the baseball. (common sense)

- But what if Luke instead turned on a flashlight?



What is the speed of light according to Luke?

According to John?

- According to Luke, the speed of light is c.
  According to John, the speed of light is also c.
  This is the <u>principle of the speed of light</u>:
  that the speed of light in a vacuum is c
  ($c = 3 \times 10^8$ m/s or 300,000 km/sec) for
  all observers, regardless of their motion
  relative to each other ~~and~~ or the speed of
  the light source.
- If you find this paradoxical, then you are thinking about it correctly!
- All that we talk about in the coming days will be derived logically and mathematically from accepting just these two principles
  1. <u>The principle of relativity</u>
  2. <u>The principle of the speed of light.</u>

- We will find that if we take the speed of light in a vacuum to be an absolute, fixed, immutable quantity, then we will have to let space and time become more flexible.

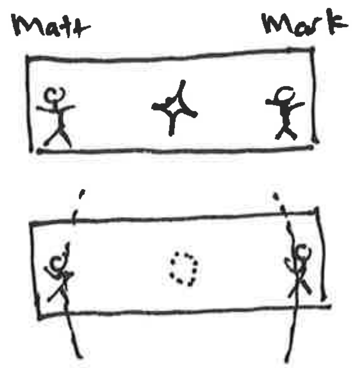
- This is the opposite approach to what Newton thought. Newton thought that space and time were fixed and immutable. That is, everyone agrees on the distance between objects, and ~~everyone~~ time proceeds at the same rate for everyone. Hence the speed of light must be flexible: it must depend on the ~~mo~~ relative motion of observers.

- Newton and Einstein simply disagreed on what is fixed (and absolute), and what is observer dependent (and hence relative.)

- I will outline a few of the important consequences of accepting the principle of relativity & the principle of the speed of light

1. The order of events may be different to observers who are in motion relative to one another. (An event is something that happens at a particular time and a particular location – such as snapping a finger or slamming a door).
2. Moving clocks tick slowly (time dilation)
3. Moving bodies are shortened (length contraction)
4. Moving objects are more massive (mass $\neq$ rest mass)

1. <u>Order of events</u>. Suppose Matthew & Mark are on opposite ends of a spaceship, equidistant from a flashbulb at the center of the ship.

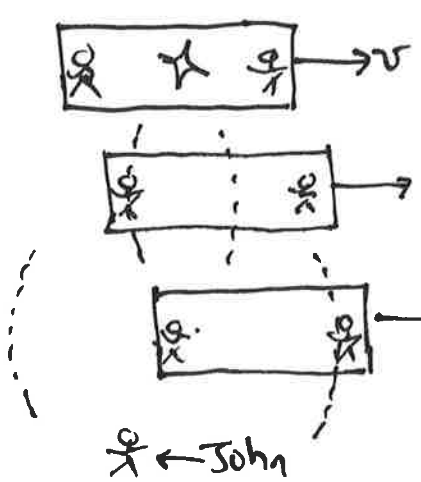


A flashbulb ignites between them.

Light pulses arrive at their locations at the same time.

Matt & Mark conclude that <u>the two events</u> (light arrives at Matt & light arrives at Mark) are <u>simultaneous</u>.

- Now, according to John (and his friends) who are floating in outerspace & see the spaceship rush by ....



Flashbulb ignites

Matt receives light pulse (event 1)

Mark receives light pulse (event 2)

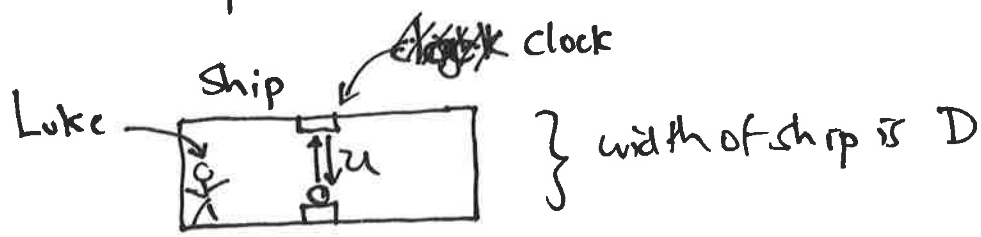
According to John, the two events occur <u>not</u> simultaneously. Event 1 occurs first, event 2 occurs second.

- A few things to notice. First, this is not an optical illusion that arises because light takes time to get to John. John could have dozens of friend astronauts who are all floating in space at rest with respect to John. They <u>all</u> see the ship going by at v. And they <u>all</u> agree that the light arrives at the position of Matthew (event 1) before the position of Mark (event 2).

- There is no way of objectively claiming that the measurements of Matthew & Mark (on the ship) are more or less reliable than the measurements of John (watching the ship go by). This is because both sets of observers are in inertial reference frames (that is: they may assume that they are at rest, and the others are in motion).

- All observers see the speed of light in vacuum moving at c compared to <u>themselves</u> (but not necessarily compared to <u>others</u>.)

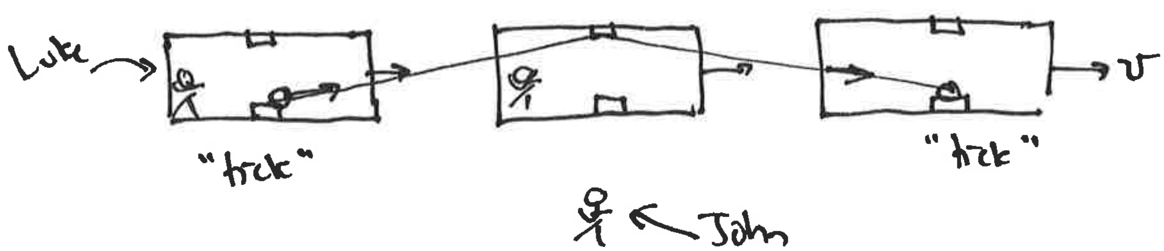
2. <u>Time dilation</u>. Consider a ship with a special clock consisting of a ball that bounces back & forth across the ship. Each time it hits the bumper on one side it "ticks".



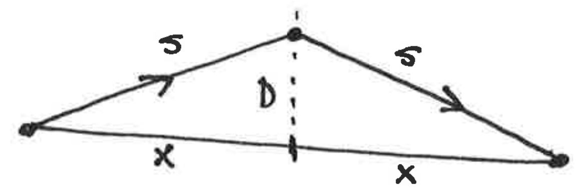
- The width of the clock is D and the speed of the ball is u. So the time between "ticks" of Luke's clock, <u>as measured by Luke</u> is

$$\Delta t_L = \frac{2D}{u}.$$

- Now if John, an astronaut floating in space, sees Luke's ship flying past, he will see the ball has to move <u>farther</u> between "ticks". But it also moves <u>faster</u>, since we can add the ship's speed v to the ball speed u:



- How much time does it take for Luke's clock to tick, according to John, who sees the clock in motion at speed v?



- The ball has to move a distance $2s$. It will be going at speed $w = \sqrt{v^2+u^2}$ (using Pythagorean theorem to add velocities of ship & ball). The distance $s = \sqrt{x^2+D^2}$ where x is the distance the ship moves in a half-tick : $x = \frac{\Delta t_J}{2} \cdot v$

- Lets plug these in and solve for Δt_J

$$\Delta t_J = \frac{2s}{w} = \frac{2\sqrt{x^2+D^2}}{\sqrt{v^2+u^2}} = \frac{2\sqrt{\left(\frac{\Delta t_J}{2}\cdot v\right)^2+D^2}}{\sqrt{v^2+u^2}}$$

$$\Delta t_J^2 = \frac{4\left(\frac{\Delta t_J^2 v^2}{4} + D^2\right)}{v^2+u^2} \quad \text{(squaring both sides)}$$

$$\Delta t_J^2 (v^2+u^2) = \Delta t_J^2 v^2 + 4D^2$$

$$\Delta t_J^2 (v^2+u^2-v^2) = 4D^2$$

$$\Delta t_J = \frac{2D}{u} = \Delta t_{Luke}$$

- Notice that, if we add the ship's speed to the ball's speed, according to ordinary velocity addition rules, then the ball has to move farther, but it's just going fast enough that both John & Luke agree on the time between ticks of Luke's clock.

- But now consider a special light clock, where a light pulse (instead of a ball) bounces back & forth between mirrors.

- Now Luke, aboard the ship, sees his clock tick with spacing between ticks

$$\Delta t_L = \frac{2D}{c}$$

- What does John measure? Since now we do not add the speed of the ship to the speed of light, we get..

$$\Delta t_J = \frac{2s}{c} = \frac{2\sqrt{x^2+D^2}}{c}$$

$$= \frac{2\sqrt{\left(\frac{\Delta t_J \cdot v}{2}\right)^2 + D^2}}{c}$$

$$\Delta t_J^2 = \frac{4}{c^2}\left(\frac{\Delta t_J^2 v^2}{4} + D^2\right)$$

$$\Delta t_J^2 = \frac{\Delta t_J^2 \, v^2}{c^2} + \frac{4D^2}{c^2}$$

$$\Delta t_J^2\left(1 - \frac{v^2}{c^2}\right) = \frac{4D^2}{c^2}$$

$$\Delta t_J^2 = \frac{4D^2}{c^2} \frac{1}{\left(1-\frac{v^2}{c^2}\right)}$$

$$\Delta t_J = \frac{2D}{c} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Delta t_J = \Delta t_L \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

- Notice now that $\Delta t_J \neq \Delta t_L$

The time between ticks of Luke's clock as measured by John is not the same as the value measured by Luke!

- Conclusion: <u>moving clocks tick slowly</u>

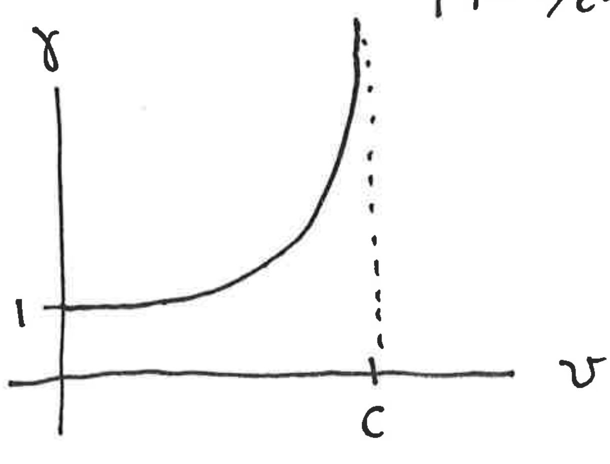
This is called <u>time dilation</u>. The general formula for time dilation is

$$\Delta t = \Delta t_0 \, \gamma$$

where Δt_0 = the time between 2 events as measured by an observer who sees the events happening at the same location (in our case, Luke)

Δt = the time between the same 2 events as measured by an observer who is in motion relative to the other observer (who measures Δt_0)

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$ = gamma factor (always > 1)



For small v, $\gamma \approx 1$

For v close to c, $\gamma \to \infty$

- Time dilation example problem: a space-craft travels @ 0.6c past an astronaut. A clock on the ship ticks once per second according to an observer on the ship. How often (time between ticks) does the clock tick according to an observer who sees the ship racing by?

$$\Delta t = \gamma \Delta t_0$$

$$= \frac{1}{\sqrt{1-\left(\frac{0.6c}{c}\right)^2}} (1 \text{ sec})$$

$$= \frac{1}{\sqrt{1-(0.6)^2}} (1 \text{ sec})$$

$$= \frac{1}{\sqrt{1-.36}} (1 \text{ sec})$$

$$= \frac{1}{0.8} (1 \text{ sec})$$

$$\Delta t = 1.25 \text{ sec}$$

- one more comment on time dilation:

  time dilation is not unique to light-clocks. It even applies to ball-clocks. As it turns out we cannot simply add the speed of the ship to the speed of the ball ($w = \sqrt{v^2 + u^2}$) in such a simple way. We'll have to come back & re-evaluate this later.

- The point for now (if you'll accept it...) is that all clocks (including your biological clocks, your heart rate, your wrist-watch, the radioactive decay of elements, etc) tick slowly when in motion.

3. <u>Relativistic</u> <u>length</u> <u>contraction</u>. How is the length of an object (such as a train car) measured? There are 2 perfectly valid practical methods

<u>method</u> 1) an observer at rest with respect to the object places a stationary ruler along the object.

<u>method</u> 2) an observer who sees the object rushing past at speed v measures how much time elapses between when the front & back of the object pass his location.

<u>example</u> • Luke is riding on a train which is rushing past John at speed v.

Luke, who is at rest with respect to the train, measures the length of the train (using method 1) to be 10 meters. $L_L = 10m$

- John, who sees the train rushing past at speed v, measures the time interval between when the front & back of the train pass. That is, he uses method 2 to time the interval Δt_J between event 1 (front of train passes) and event 2 (back of train passes). He concludes that

$$L_J = v \, \Delta t_J$$

- Now: does $L_J \stackrel{?}{=} L_L$. That is, do they agree on the length of the train? To compare these, we could ask Luke to find the time interval between when John (moving at speed v according to Luke) passes the front of the train (event 1) and the back of the train (event 2). The interval that Luke obtains is $\Delta t_L = \frac{L_L}{v}$

- But from our time dilation formula (pg 14)

$$\Delta t_L = \gamma \, \Delta t_J$$

(since John is measuring Δt_0 in this case).

- Combining these formulas:

$$L_J = v\,\Delta t_J$$
$$= v\,\frac{\Delta t_L}{\gamma}$$
$$= \frac{1}{\gamma}\,v\cdot\Delta t_L$$
$$L_J = \frac{L_L}{\gamma}$$

So Luke & John measure different lengths!

- Since $\gamma > 1$, John measures that the train is <u>shorter</u> than 10 meters.

- The general formula for relativistic length contraction is

$$L = \frac{L_0}{\gamma}$$

where L_0 = the length of an object as measured by an observer who is at rest with respect to the object being measured

L = the length of the same object as measured by an observer in motion compared to the other observer

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Notice that, in the previous example, Luke measured the length L_0 of the train, but John measured Δt_0, the time between the 2 events that allowed them to measure the length of the train.

- Length contraction & time dilation example

A spaceship travels at a uniform speed between Earth & alpha-centauri. The speed of the ship compared to earth is 0.95c. The distance between earth & alpha centauri, as measured by an observer at rest with respect to the earth (and also alpha-centauri) is 4.3 light years (how far light travels in 1 year). Question: how much do (i) an earth-bound observer and (ii) a spaceship-bound astronaut age between the two events (event 1: spaceship leaves earth)
(event 2: spaceship arrives @ alpha-centauri).

Notice that the earth-bound observer measures the length L_0 between earth & α-centauri. and the astronaut measures Δt_0, the time between when earth leaves him & α-centauri arrives at ~~them~~ him.

$$L_0 = (4.3\,\text{years})\,c$$

$$\Delta t = \frac{L_0}{v} = \frac{(4.3\,\text{years})\,c}{0.95\,c} = 4.5\,\text{years}$$

$$\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{4.5\,\text{years}}{1/\sqrt{1-\left(\frac{0.95c}{c}\right)^2}} = \frac{4.5\,\text{yrs}}{3.2} = 1.4\,\text{yrs.}$$

- So the astronaut sees 1.4 years elapse during the trip, whereas the earth-bound observer sees 4.5 years elapse. What is the distance traveled? According to the earth-bound observer, the ship goes 4.3 light years. According to the astronaut, the trip is

$$L = \frac{L_0}{\gamma} = \frac{(4.3\,\text{years})\cdot c}{3.2} = 1.3\,\text{light years.}$$

- Notice that the two observers disagree on the distances & the time intervals. But they agree on the relative speed v and the speed of light c.

4. <u>Relativistic mass increase</u> — energy, in all of its forms, behaves as though it had mass.

- Light, for example, is known to have energy (it can heat up targets) but it was thought to have zero mass. Nonetheless, it acts as though it has mass. How? It carries <u>momentum</u>.
- We know this because it can exert a <u>force</u> on objects. (1997 Nobel prize -- lasers can be used to sort objects by exerting a force.)
- Einstein's formula $\boxed{E = mc^2}$ captures this <u>equivalence</u> of energy & mass.
- Let's first do an example of why this is important, then come back & talk about this formula more

Nuclear fusion example

$$^{2}_{1}H + ^{3}_{1}H \longrightarrow ^{4}_{2}He + ^{1}_{0}n$$

deuterim + tritium $\longrightarrow$ helium-4 + neutron

- Notice that 2 protons go in to this nuclear reaction

  and 3 neutrons " " " " " "

  and the same number of protons & neutrons go out.

- 1 atomic mass unit $u = 1.6605 \times 10^{-27}$ kg

- The mass of the reactants is greater than the mass of the products. This is because some of the mass of the reactants shows up as kinetic energy of the products!

reactants	mass	product	mass
$^{2}_{1}H$	2.014u	$^{4}_{2}He$	4.003u
$^{3}_{1}H$	3.016u	$^{1}_{0}n$	1.009u
Total	5.030u	Total	5.012u

ΔE = enrgy equivalent of difference in mass of reactants & products

$= \Delta m c^2 = (5.030u - 5.012u)\, c^2$

$= 2.69 \times 10^{-13}$ Joules

- So $\approx 2.7 \times 10^{-13}$ Joules of kinetic energy is created from the mass of the reactants
- For 1 mole of reactants, this gives

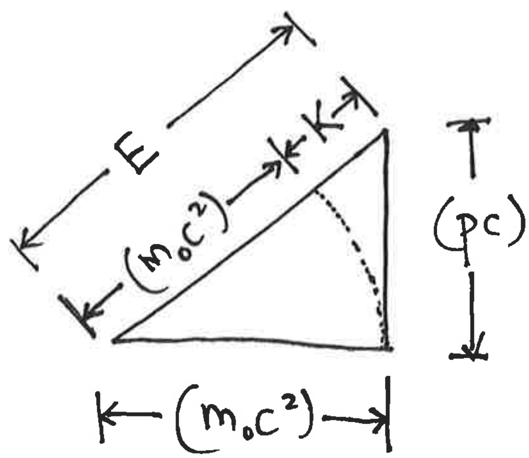
$$2.7 \times 10^{-13} \text{ Joules} \times 6.02 \times 10^{23} \approx 18 \times 10^{10} \text{ Joules}$$

- By comparison, one mole of dynamite releases just 10^{4} Joules.
- So a nuclear fission reaction creates over a million times more energy than an equivalent (mass of) chemical reaction.
- This makes nuclear reactions very efficient in producing power!

- Comparing Newtonian dynamics with Relativistic dynamics

Newton		Einstein
$F = \frac{dp}{dt}$	Newton's 2nd law	$F = \frac{dp}{dt}$
$p = m_0 v$	momentum	$p = \gamma m_0 v$
$\underset{\text{mass}}{m} = \overset{\text{rest mass}}{m_0}$	mass	$\underset{\text{mass}}{m} = \gamma \overset{\text{rest mass}}{m_0}$
$K = \frac{1}{2} m v^2$	kinetic energy	$K = E - m_0 c^2$
$K = \frac{p^2}{2m}$	Relation between energy & momentum	$E^2 = (pc)^2 + (m_0 c^2)^2$

- A useful diagram relating energy & momentum for relativistic dynamics.



- In relativistic dynamics, all forms of potential energy (gravitational, chemical, thermal) show up in the rest energy $(m_0 c^2)$. The kinetic energy shows up as K.