EX 32.2 (Relativistic energy)

a) The rest mass of an electron is $m_0 = 9.1e{-31}$ kg.

So its rest energy is $E_0 = m_0 c^2$

$\boxed{E_0 = 8.2e{-14} \text{ Joules}}$

when it is moving at $0.995c$, $\gamma = 10$, so

$E = \gamma m_0 c^2 = \boxed{8.2e{-13} \text{ Joules}}$

Its kinetic energy is thus $KE = E - E_0 = \boxed{7.4e{-13}\,J}$

Classically, $KE = \frac{1}{2}mv^2 = \boxed{4\times 10^{-14}\ J}$

These differ by a factor of about $\boxed{18.}$

b) Since $\underbrace{p = mv \text{ and } E = \frac{1}{2}mv^2}$

$\boxed{E = p^2/2m}$ ← classicaly

Relativistically (i) $E = mc^2 = \gamma m_0 c^2$ (ii) $P = \gamma m_0 v$ (iii) $\gamma^2 = \frac{1}{1 - v^2/c^2}$ } $E^2 = \gamma^2 m_0^2 c^4$ (i) $p^2 = \gamma^2 m_0^2 v^2$ (ii) $\gamma^2 - \frac{\gamma^2 v^2}{c^2} = 1$ (iii)

Combining these { $\gamma^2 m_0^2 c^4 - \gamma^2 v^2 m_0^2 c^2 = m_0^2 c^4$

$E^2 - p^2c^2 = (m_0 c^2)^2$

or $\boxed{E^2 = (pc)^2 + (m_0 c^2)^2}$

Ex 32.2 (cont'd)

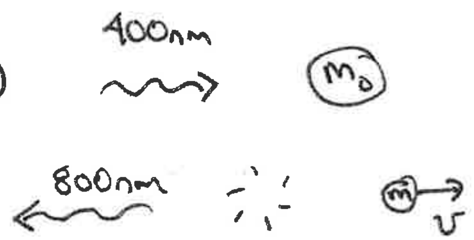
c) For a massless particle of wavelength λ,

$$E^2 = (pc)^2 + (m_o c^2)^2$$

$$= \left(\frac{hc}{\lambda}\right)^2 + 0$$

or $\boxed{E = \frac{hc}{\lambda}}$ ← photon energy

d)



initial: $p_i = +\frac{h}{\lambda_i}$, $K_i = \frac{hc}{\lambda_i}$

final: $p_f = \frac{h}{\lambda_f}$, $K_f = \frac{hc}{\lambda_f}$

The photon's momentum is changed by $\Delta p = p_f - p_i$. So this momentum is gained by the particle. Also, the photon's KE is changed by $\Delta K = K_f - K_i$. So this KE is gained by the particle. Using

$E^2 = (cp)^2 + (m_o c^2)$ and $E = m_o c^2 + KE$ we can show that $(cp)^2 = (KE)^2 + 2(KE)\cdot m_o c^2$

Now $(c\Delta p)^2 = (\Delta KE)^2 + 2(\Delta KE)\cdot m_o c^2$

or $$m_o = \frac{(c\Delta p)^2 - (\Delta KE)^2}{2(\Delta KE)c^2}$$

$$\boxed{m_o = 1.1 \times 10^{-35}\ \text{kg}}$$