

ASG v1 Ex 18.1 (Angular width)

Mercury dime



$$D_D = 17.9 \text{ mm}$$

America Eagle



$$D_E = 40.6 \text{ mm}$$

} actual diameters.

Here we will try to describe how large these coins appear when held at a distance of 90 cm from one's eye. ($R = 900 \text{ mm}$)

a) The areas of the two coins are

$$\text{dime: } A_D = \pi \left(\frac{D_D}{2} \right)^2 = 251.6 \text{ mm}^2$$

$$\text{eagle: } A_E = \pi \left(\frac{D_E}{2} \right)^2 = 1295 \text{ mm}^2$$

The angular width of the coins are given by $D = \Theta R$

$$\Theta_D = \frac{17.9 \text{ mm}}{900 \text{ mm}} = 0.0199 \text{ rad} = 1.14 \text{ deg.}$$

$$\Theta_E = \frac{40.6 \text{ mm}}{900 \text{ mm}} = 0.0451 \text{ rad} = 2.58 \text{ deg.}$$

The solid angle (in steradians) is the fraction of the area of a sphere that an object held at distance R occupies

$$\text{S.A.} = \frac{\text{Area of object}}{\text{Area of sphere at object's distance}}$$

So the solid angles of the cones are

$$SA_D = \frac{251.6 \text{ mm}^2}{4\pi (900 \text{ mm})^2} = 2.47 \times 10^{-5} \text{ steradians}$$

$$SA_E = \frac{1295 \text{ mm}^2}{4\pi (900 \text{ mm})^2} = 12.7 \times 10^{-5} \text{ steradians}$$

b) In order to magnify the disc so that it looks to be the same size as the eagle, one would need to employ a power of

$$\frac{1295 \text{ mm}^2}{251.6 \text{ mm}^2} = 5.15$$

This is a comparison of areas. If we want to compare apparent diameters, we would use

$$\frac{40.6}{17.9} = 2.27 \text{ magnification.}$$

The magnification of a telescope can be found by dividing the focal length of the telescope (from primary mirror or lens) by the focal length of the eyepiece lens. So if $f_{\text{primary}} = 2032 \text{ mm}$, we'd use $f_e = 895 \text{ mm}$