ASGv1 Ex 15.3 (Platonic Solids)

a) There are 5 platonic solids: the tetrahedron (4 sides), the cube (6 sides), the octahedron (8), the dodecahedron (12) and the icosahedron (20).

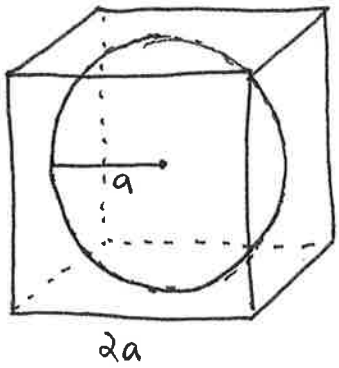
b) For an interesting disussion of the history and significance of the platonic solids - stretching back to the work of Plato ca. 350 B.C. - see Wolfram's Math World entry on Platonic solids. Briefly, the Platonic solids are "convex polyhedra with equivalent faces composed of congruent convex regular polygons." The faces of the cube are squares; of the dodecahedron are pentagons; of the rest are triangles.

Soln EX 15.5 (Platonic Solids)

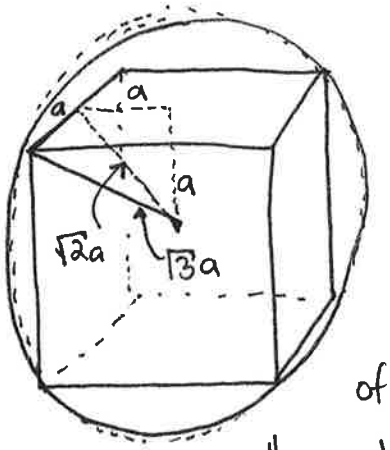
a) There are 5 platonic solids: the tetrahedron (4 sides), the cube (6 sides), the octahedron (8), the dodecahedron (12) and the icosahedron (20).

b) For an interesting discussion of the history and significance of the Platonic solids - stretching back to the work of Plato ca. 350 B.C. - see Wolfram's Math World entry on Platonic Solids. Briefly, the Platonic solids are "convex polyhedra with equivalent faces composed of congruent convex regular polygons." The faces of the cube are squares, of the dodecahedron are pentagons, of the rest are triangles.

c) The problem asks for the ratio of the radii of the spheres circumscribed and inscribed in a cube. Consider a cube of side length $2a$.



The inscribed sphere has radius a, since it touches each face of the cube.



The circumscribed sphere has radius $\sqrt{3}a$. This can be seen because a line drawn from the center of the sphere to an edge of the cube has length $\sqrt{a^2+a^2} = \sqrt{2}a$

Therefore a line drawn from the center of the sphere to a vertex of the cube has length $\sqrt{a^2 + (\sqrt{2}a)^2} = \sqrt{a^2 + 2a^2} = \sqrt{3}a$

The ratio of the radii of the circumscribed and inscribed spheres is therefore $\frac{\sqrt{3}a}{a} = \sqrt{3}$.

Interestingly, the ratio of the distance between Saturn and the sun and the distance between Jupiter and the sun is quite close to $\sqrt{3}$. Using modern data for the mean (average) distance to the sun: Jupiter is 7.87×10^8 km and Saturn is 1.43×10^9 km, the ratio is $\frac{1.43 \times 10^9}{7.87 \times 10^8} = 1.82$ whereas $\sqrt{3} = 1.73$. Kepler viewed this <u>not</u> as coincidence, but rather as a plan which God used to lay out the solar system. Today, scientists view this as a coincidence. This raises the question: how close would it have to be to $\sqrt{3}$ before scientists would not view it as a coincidence?